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APPLICATION OF THE PERTURBATION METHOD TO CONSTRUCT A REFINED COMPACT MODEL OF A THERMOELECTRIC ELEMENT WITH TEMPERATURE-DEPENDENT PARAMETERS

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Abstract. The paper presents an asymptotically substantiated compact model of the Peltier element. The problem of stationary temperature distribution in a one-dimensional thermoelectric medium with temperature-dependent physical parameters is considered. A direct asymptotic approximation is constructed under the assumption that the ratio of the temperature difference at the boundaries of the Peltier element to the mean absolute temperature of the module is a small value. Expressions for heat fluxes on the absorbing and radiating sides with a nonlinear dependence on the applied current and boundary temperatures are obtained. A method of synthesis based on the obtained solution of a compact system model of a thermoelectric module is proposed. A numerical example is used to compare the obtained model with the classical model with averaged material parameters. It is shown that the heat fluxes of the two models take different values at sufficiently large electric currents. Promising areas of using the proposed new analytical model of the Peltier element in industrial problems are discussed.

Keywords: Peltier battery, Matlab, Simscape, direct asymptotic expansion method, system-level modeling, reduced order modelling

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ПРИМЕНЕНИЕ МЕТОДА ВОЗМУЩЕНИЙ ДЛЯ ПОСТРОЕНИЯ УТОЧНЕННОЙ КОМПАКТНОЙ МОДЕЛИ ТЕРМОЭЛЕКТРИЧЕСКОГО ЭЛЕМЕНТА С ТЕРМОЗАВИСИМЫМИ ПАРАМЕТРАМИ

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Аннотация. В работе выполняется построение асимптотически обоснованной компактной модели элемента Пельтье. Рассматривается задача о стационарном распределении температуры в одномерной термоэлектрической среде с физическими параметрами, зависящими от температуры. Построено прямое асимптотическое приближение в предположении о том, что отношение разности температур на границах элемента Пельтье к средней абсолютной температуре модуля является малой величиной. Получены выражения для тепловых потоков на поглощающей и излучающей сторонах с нелинейной зависимостью от приложенного тока и граничных температур. Предложена методика синтеза на основе полученного решения компактной системной модели термоэлектрического модуля. На численном примере проведено сравнение полученной модели с классической моделью с осредненными параметрами материала. Показано, что тепловые потоки двух моделей принимают разные значения при достаточно больших электрических токах. Обсуждаются перспективные направления использования предложенной новой аналитической модели элемента Пельтье в задачах промышленности.

Ключевые слова: батарея Пельтье, Matlab, Simscape, метод прямого асимптотического разложения, системное моделирование, модели пониженного порядка

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Introduction

Thermoelectric modules based on the Peltier effect are an important part of modern thermal stabilization and cooling systems [1–11]. Such systems are widely used, for example, to create miniature refrigeration units and sensors that register temperature differences [11].

In practice, constant physical parameters of the object are used for analytical estimations of dependence of the main parameters of thermoelectric modules (cooling capacity, maximum temperature difference between module surfaces, etc.) on the boundary values of temperatures and heat fluxes. The obtained estimates under this assumption do not always coincide with experimental data [1–3, 5, 6, 12, 13]. Taking into account the dependence of material parameters on temperature can strongly change the module performance and its output parameters with respect to similar values calculated with constant material parameters. For a more accurate description of the battery parameters or design (selection of material properties of p- and n- types) of the battery, a correct mathematical model is required.

A one-dimensional analytical model of the stationary state of a thermoelectric element is investigated in [1, 2]. Temperature distributions along the Peltier element as a function of current density under boundary conditions of the first and second kind are obtained. In [5–9], the methodology of modelling of Peltier elements based on the system approach with application of compact models is proposed. Numerical dependences of temperatures and heat fluxes of the Peltier element are obtained based on the solution of the linear boundary value problem with constant material parameters. The presented approach is useful and practical, since only manufacturers' data sheets are used to obtain the system model parameters. In [10, 11], expressions for the optimal current value, at which the maximum temperature difference occurs in the Peltier element, are given. A nonlinear equation for temperature distribution taking into account temperature-dependent material properties is obtained. The influence of temperature-dependent physical parameters on the temperature distribution along the Peltier element is discussed. It is shown that the heat conduction model with temperature-dependent properties significantly refines the temperature distribution in comparison with the model with averaged material parameters. In [14–19], mathematical models of unsteady pulse impact on a thermoelectric element leading to the so-called supercooling effect were constructed and investigated.

In this paper, we consider the problem of finding an approximate analytical solution to the problem of stationary temperature distribution for a Peltier element, when temperature-dependent physical properties of the material are taken into account. Using the mathematical apparatus of perturbation methods, modified expressions of heat fluxes at the boundaries of the Peltier element are determined. A compact system model of the Peltier element is proposed, taking into account the obtained refined thermoelectric characteristics.

Mathematical model

The stationary thermoelectric state of a one-dimensional medium is described by the equation [1–3]:

$$\kappa(T)\frac{d^2T}{dx^2}+\frac{d\kappa(T)}{dT}\left(\frac{dT}{dx}\right)^2-j_0T\frac{dS(T)}{dT}\frac{dT}{dx}=-\frac{j_0^2}{\sigma(T)}; \tag{1a}$$

$$T(x=0)=T_L,\ T(x=L)=T_R, \tag{1b}$$

where T is the temperature; (T), $\sigma(T)$, $S(T)$ are the temperature-dependent thermal conductivity, electrical resistivity and Seebeck coefficients, respectively; j_0 is the current density, x is the spatial coordinate.

A schematic representation of the problem under consideration is presented in Fig. 1.

An example of the dependences of experimental parameters $\kappa(T)$, $\sigma(T)$, $S(T)$ for BiSbTe n- $\left(\kappa_n^T(T),\ \sigma_n^T(T),\ S_n^T(T)\right)$ and p- $\left(\kappa_p^T(T),\ \sigma_p^T(T),\ S_p^T(T)\right)$ types in the temperature range from 300 to 500 K is given in Fig. 2, 3 [19]. The experimental data were approximated by parabolic polynomials, where the lower index n, p defines the material properties calculated for n- and p-type BiSbTe material.

Expressions for $\kappa_{n,p}^T(T)$, $\sigma_{n,p}^T(T)$, $S_{n,p}^T(T)$ are written as:

$$\begin{aligned}\kappa_{n,p}^T(T)&=\kappa_0^{n,p}+\kappa_1^{n,p}T+\kappa_2^{n,p}T^2;\\ \sigma_{n,p}^T(T)&=\sigma_0^{n,p}+\sigma_1^{n,p}T+\sigma_2^{n,p}T^2;\\ S_{n,p}^T(T)&=S_0^{n,p}+S_1^{n,p}T+S_2^{n,p}T^2.\end{aligned} \tag{2}$$

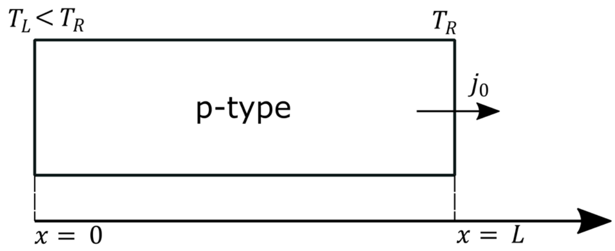

Fig. 1. One-dimensional model of a p-type element (constant cross-section A)

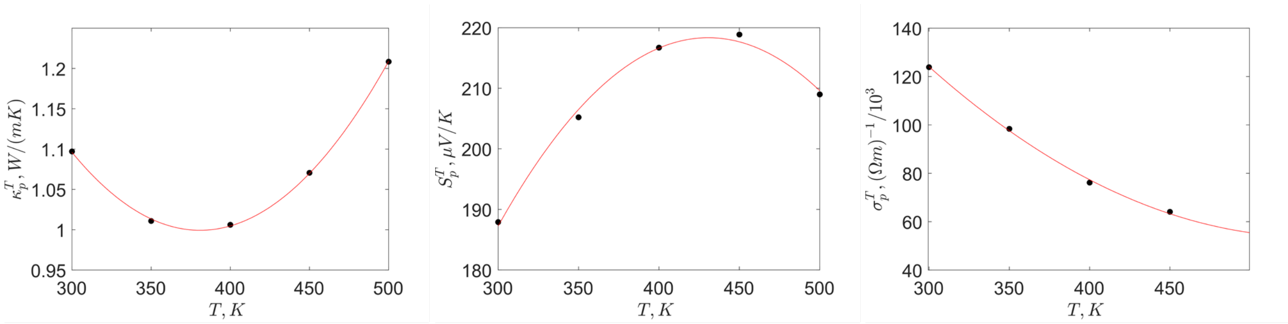

Fig. 2. Polynomial approximation (solid line) of experimental data
for n-type $\kappa_p^T(T)$, $\sigma_p^T(T)$, $S_p^T(T)$ (points) of BiSbTe material

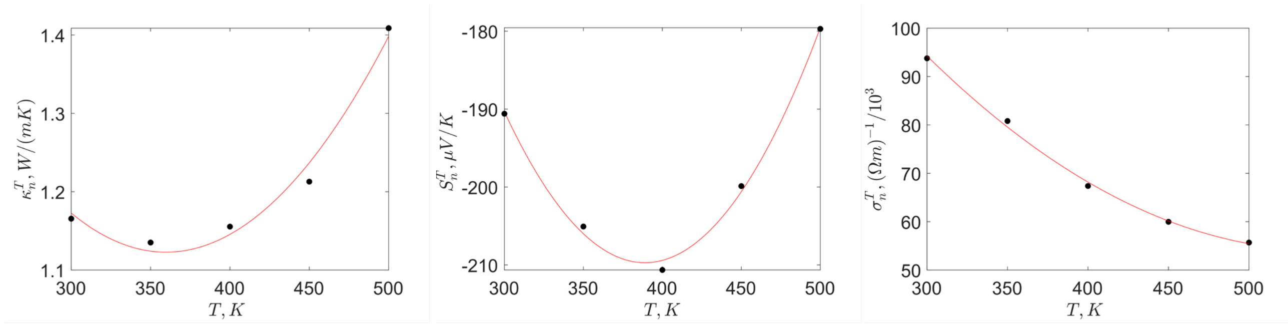

Fig. 3. Polynomial approximation (solid line) of experimental data
for n-type $\kappa_n^T(T)$, $\sigma_n^T(T)$, $S_n^T(T)$ (points) BiSbTe material

Based on the representation of physical parameters by expressions (2), equation (1) is characterized by a degree nonlinearity with respect to the variable T. The analytical representation of the material parameters $\kappa_{n,p}^T$, $\sigma_{n,p}^T$, $S_{n,p}^T$ (2) allows us to apply asymptotic methods [20, 21] to find an approximate solution for the temperature distribution (1a).

Let us introduce dimensionless quantities:

$$u = \frac{T - \hat{T}}{\hat{T}}, \ \xi = \frac{x}{L}, \ \hat{T} = \frac{T_L + T_R}{2}. \tag{3}$$

Then equation (1) is rewritten in the form:

$$\kappa(u)\frac{d^2u}{d\xi^2}+\frac{d\kappa(u)}{du}\left(\frac{du}{d\xi}\right)^2-j_0L(u+1)\frac{dS(u)}{du}\frac{du}{d\xi}=-\frac{(j_0L)^2}{\hat{T}}\frac{1}{\sigma(u)'};\tag{4a}$$

$$u(\xi=0)=\frac{T_L-\hat{T}}{\hat{T}},\ u(\xi=1)=\frac{T_R-\hat{T}}{\hat{T}},\tag{4b}$$

where u is the small and finite relative temperature change; $\kappa(u),\ \sigma(u),\ S(u)$ (lower indices n, p omitted hereafter) are defined as:

$$\kappa(u)=\hat{\kappa}_0+\hat{\kappa}_1u+\hat{\kappa}_2u^2,\ \sigma(u)=\hat{\sigma}_0+\hat{\sigma}_1u+\hat{\sigma}_2u^2,\ S(u)=\hat{S}_0+\hat{S}_1u+\hat{S}_2u^2,\tag{5}$$

and the coefficients $\hat{\kappa}_0,\ \hat{\kappa}_1,\ \hat{\kappa}_2,\ \hat{\sigma}_0,\ \hat{\sigma}_1,\ \hat{\sigma}_2,\ \hat{S}_0,\ \hat{S}_1,\ \hat{S}_2$ take the form of:

$$\begin{gathered}\hat{\kappa}_0=\kappa_0+\kappa_1\hat{T}+\kappa_2\hat{T}^2,\ \hat{\kappa}_1=\kappa_1\hat{T}+2\kappa_2\hat{T}^2,\ \hat{\kappa}_2=\kappa_2\hat{T}^2;\\ \hat{\sigma}_0=\sigma_0+\sigma_1\hat{T}+\sigma_2\hat{T}^2,\ \hat{\sigma}_1=\sigma_1\hat{T}+2\sigma_2\hat{T}^2,\ \hat{\sigma}_2=\sigma_2\hat{T}^2;\\ \hat{S}_0=S_0+S_1\hat{T}+S_2\hat{T}^2,\ \hat{S}_1=S_1\hat{T}+2S_2\hat{T}^2,\ \hat{S}_2=S_2\hat{T}^2.\end{gathered}\tag{6}$$

In practice, u is often a small value [10, 11, 20], the temperature difference across the Peltier element is of the order of a few tens of degrees, while the average temperature is of the order of hundreds of degrees Kelvin. Problem (4), together with equations (5), forms a second order nonlinear differential equation, which has no analytical solution. To evaluate the solution of this problem, it is assumed that u is a small value, and the asymptotic method of direct expansion is applied [20–22].

Asymptotic solution of the one-dimensional thermoelectric problem

To find a uniformly suitable solution of the third order of smallness, we decompose u into the following series:

$$u(\xi,\varepsilon)=\varepsilon u_0(\xi)+\varepsilon^2u_1(\xi)+\varepsilon^2u_2(\xi)+\ \dots,\tag{7}$$

where ε is a small parameter.

Let us consider the term in the right part of the equation (4a), for this purpose we divide equation (4) by the value κ_0, then:

$$\frac{(j_0L)^2}{\hat{T}}\frac{1}{\kappa_0\sigma}=\frac{I^2R\Theta}{\hat{T}}=\frac{dT}{\hat{T}},\tag{8}$$

where $I=j_0A$ is an electric current through the element, $R=\frac{1}{\sigma}\frac{L}{A}$ is an electric resistance, $\Theta=\frac{1}{\kappa_0}\frac{L}{A}$ is a thermal resistance of the element, dT is a temperature difference in the element at the heat flux equal to the electric power I^2R.

From a practical point of view, it makes sense to apply a thermoelectric element only in the region, where the heat flux from the Joule–Lenz effect is significantly smaller in comparison with the basic Fourier heat flux. In this regard, we will assume that the summand in the right-hand side can be attributed to quantities of the order ε. Let us take into account the smallness of the heat fluxes caused by the

Thomson effect for the class of materials under consideration $\left(\frac{j_0 L\hat{S}_1}{\hat{\kappa}_0} \sim 10^{-1}\right)$. We will assume that the summand with the Seebeck coefficient of equation (4a) does not enter the generating problem, that is, $j_0 L\frac{dS}{du} \sim \varepsilon$.

Substituting the equations of expansion (7) into equation (4) and equating the coefficients at the same powers of ε, we obtain:

ε

$$u_0'' = -\frac{j_0^2 L^2}{\hat{T}\hat{\kappa}_0\hat{\sigma}_0}, \tag{9a}$$

$$u_0(\xi=0) = \frac{T_L - \hat{T}}{\hat{T}},\ u_0(\xi=1) = \frac{T_R - \hat{T}}{\hat{T}}; \tag{9b}$$

ε^2

$$u_1'' = \frac{j_0 L S_1}{\hat{\kappa}_0}u_0' - \left(\frac{\hat{\kappa}_1}{\hat{\kappa}_0} + \frac{\hat{\sigma}_1}{\hat{\sigma}_0}\right)u_0 u_0'' - \frac{\hat{\kappa}_1}{\hat{\kappa}_0}u_0'^2, \tag{10a}$$

$$u_1(\xi=0) = 0,\ u_1(\xi=1) = 0; \tag{10b}$$

ε^3

$$\begin{gathered} u_2'' = \frac{j_0 L S_1}{\hat{\kappa}_0}u_1' - \left(\frac{\hat{\kappa}_1}{\hat{\kappa}_0} + \frac{\hat{\sigma}_1}{\hat{\sigma}_0}\right)\left(u_0 u_1'' + u_1 u_0''\right) - \\ -\left(\frac{\hat{\kappa}_2}{\hat{\kappa}_0} + \frac{\hat{\sigma}_2}{\hat{\sigma}_0} + \frac{\hat{\kappa}_1}{\hat{\kappa}_0}\frac{\hat{\sigma}_1}{\hat{\sigma}_0}\right)u_0^2 u_0'' - \left(\frac{2\hat{\kappa}_2}{\hat{\kappa}_0} + \frac{\hat{\kappa}_1}{\hat{\kappa}_0}\frac{\hat{\sigma}_1}{\hat{\sigma}_0}\right)u_0 u_0'^2 - \\ -\frac{2\hat{\kappa}_1}{\hat{\kappa}_0}u_0' u_1' + \frac{j_0 L}{\hat{\kappa}_0}\left(\left(1 + \frac{\hat{\sigma}_1}{\hat{\sigma}_0}\right)S_1 + 2S_2\right)u_0 u_0', \end{gathered} \tag{11a}$$

$$u_2(\xi=0) = 0,\ u_2(\xi=1) = 0. \tag{11b}$$

The first approximation problem given by equations (9) is similar to the thermoelectric problem with constant (averaged) material properties:

$$u_0 = -\frac{c_0}{2}\xi^2 + c_1\xi + c_2, \tag{12}$$

where

$$c_0 = \frac{j_0^2}{\hat{\kappa}_0\hat{\sigma}_0}\frac{L^2}{\hat{T}},\ c_1 = \frac{c_0}{2} + \frac{T_R - T_L}{\hat{T}},\ c_2 = \frac{T_L - \hat{T}}{\hat{T}}. \tag{13}$$

By substituting the solution (11) into the boundary value problem (9), it is possible to obtain the solution for u_1. Any further refinements of the solution become more complex in structure, however, their explicit form can be obtained using computer algebra methods [23] and then transferred to the simulation environment for direct calculations or optimization procedures. For typical parameter values, the constructed solutions converge to a direct numerical solution quickly: a two- or three-term approximation can satisfy the practical needs. In the numerical example (see Section 4), the maximum relative error with respect to the direct numerical solution of the nonlinear boundary value problem realized using the Matlab bvp4c built-in function [23] is 0.69% and 0.042% for the two- and three-term solution, respectively. In the following calculations, the trinomial solution will be used.

Substituting (13) into equations (10, 11), we obtain expressions for temperatures u_1 and u_2. Due to the adopted system of notations, $\varepsilon = 1$ [21]. In this case, the equation for the dimensionless temperature u is written in the form of:

$$u = d_1\xi^8 + d_2\xi^7 + d_3\xi^6 + d_4\xi^5 + d_5\xi^4 + d_6\xi^3 + d_7\xi^2 + d_8\xi + d_9, \tag{14}$$

where the constants d_i, $i = \overline{1,9}$ are determined from equations (9–11).

The heat flux profile $q = q(u,\xi)$ can be calculated as [1, 2]:

$$q(u,\xi) = -\frac{\hat{T}}{L}\kappa(u)u' + j_0\hat{T}S(u)(u+1). \tag{15}$$

Consider the values of heat flux q on the left q_L and right q_R sides of the Peltier element.

From equation (15), taking into account (3), (13), the heat fluxes q_L, q_R have the following form:

$$\begin{aligned} q_L &= q\left(\frac{T_L-\hat{T}}{\hat{T}},0\right) = \frac{\hat{T}}{L}\kappa_L u'(0) + j_0 S_L T_L, \\ q_R &= q\left(\frac{T_R-\hat{T}}{\hat{T}},1\right) = \frac{\hat{T}}{L}\kappa_R u'(1) + j_0 S_R T_R, \end{aligned} \tag{16}$$

or:

$$\begin{aligned} q_L &= -\frac{d_8\kappa_L}{2L}(T_R+T_L) + j_0 S_L T_L, \\ q_R &= -\frac{(8d_1+7d_2+6d_3+5d_4+3d_6+2d_7+d_8)\kappa_R}{2L}(T_R+T_L) + j_0 S_R T_R, \end{aligned} \tag{17}$$

where κ_L, κ_R, S_R are κ and S calculated on the left and right sides of the Peltier element.

Equations (17) are modified expressions for heat fluxes on the radiating and absorbing sides of the Peltier element in the case of temperature-dependent material parameters. Further on the basis of expressions (15, 17) the heat fluxes on both sides of the Peltier element are estimated.

Numerical examples

The following examples show the results of calculations of temperature and heat fluxes of the thermoelectric element. The material properties taken in the calculations are given in Table 2 for p-type material. The dependence of material properties on temperature is assumed according to Fig. 2 [1, 2]. For n-type material, the constructions are similar and are not given.

Table 1

Values of thermoelectric cell parameters for BiSbTe n-type material

Variable	Value
T_L	300 K
T_R	320 K
j_0	0.83 MA/m^2
A	38.6 mm^2
L	5 mm
$\hat{\kappa}_0$	1.22 W/(m·K)
$\hat{\kappa}_1$	–0.65 W/(m·K^2)
$\hat{\kappa}_2$	1.1 W/(m·K^3)
$\hat{\sigma}_0$	1.1×10^4 (Ω·m)$^{-1}$
$\hat{\sigma}_1$	–9.93×10^4 (Ω·m)$^{-1}$/K
$\hat{\sigma}_2$	5.05×10^4 (Ω·m)$^{-1}$/K^2
$\hat{S}_0$	–1.8×10^{-4} V/K
$\hat{S}_1$	–1.6×10^{-4} V/K^2
$\hat{S}_2$	1.85×10^{-4} V/K^3
$\langle\kappa\rangle$	1.198 W/(m·K)
$\langle\sigma\rangle$	70×10^3 (Ω·m)$^{-1}$
$\langle S\rangle$	–2×10^{-4} V/K

Table 2

Values of thermoelectric cell parameters for BiSbTe p-type material

Variable	Value
$\hat{\kappa}_0$	1.17 W/(m·K)
$\hat{\kappa}_1$	–0.86 W/(m·K^2)
$\hat{\kappa}_2$	1.12 W/(m·K^3)
$\hat{\sigma}_0$	13.96×10^4 (Ω·m)$^{-1}$
$\hat{\sigma}_1$	–17.97×10^4 (Ω·m)$^{-1}$/K
$\hat{\sigma}_2$	9.4×10^4 (Ω·m)$^{-1}$/K^2
$\hat{S}_0$	1.74×10^{-4} V/K
$\hat{S}_1$	1.56×10^{-4} V/K^2
$\hat{S}_2$	–1.38×10^{-4} V/K^3
$\langle\kappa\rangle$	1.06 W/(m·K)
$\langle\sigma\rangle$	82×10^3 (Ω·m)$^{-1}$
$\langle S\rangle$	2.1×10^{-4} V/K

where $\langle\kappa\rangle$, $\langle\sigma\rangle$, $\langle S\rangle$ are averaged material parameters defined as

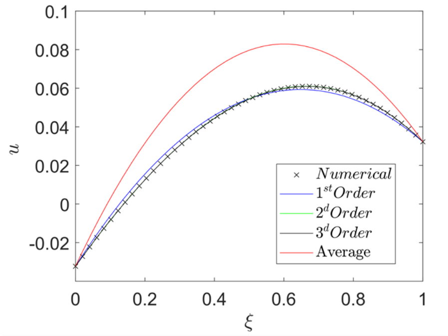

Fig. 4. Temperature profile distributions in the cases of numerical solution (crosses), results obtained with the approximated solution (blue, green and black solid lines) and the solution with averaged material parameters (red solid line)

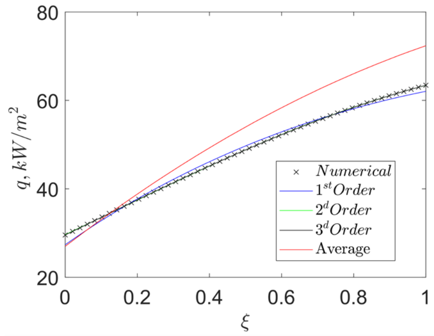

Fig. 5. Distributions of heat flux profiles in cases of numerical solution (crosses), results obtained with the approximated solution (blue, green and black solid lines) and the solution with averaged material parameters (red solid line)

$$<\kappa>=\frac{\int_{T_{\min}}^{T_{\max}}\kappa(T)dT}{T_{\max}-T_{\min}},\ <\sigma>=\frac{\int_{T_{\min}}^{T_{\max}}\sigma(T)dT}{T_{\max}-T_{\min}},\ <S>=\frac{\int_{T_{\min}}^{T_{\max}}S(T)dT}{T_{\max}-T_{\min}}, \tag{18}$$

$T_{\max}$ = 500 K, $T_{\min}$ = 300 K are maximal and minimal temperatures (see Fig. 2, 3).

We compare the temperature and heat flux distributions in the Peltier element in the case of p-type BiSbTe material between the numerical solution of equation (4) in Matlab (using bvp4c function), the asymptotic solution (9–14) and the solution of equation (4) with averaged material parameters (18) (see Fig. 4, 5).

Fig. 4, 5 show that in the case of temperature-dependent material parameters, the direct expansion method gives more accurate temperature and heat flux profiles compared to the case of averaged material parameters [1, 2]. The first asymptotic term gives better results than the solution with parameters averaged over the total temperature interval of the input data. This is explained by the fact that the physical parameters of the Peltier element are a series that decompose with respect to the average temperature of the Peltier element $\hat{T}$.

Compact Peltier battery model

To simulate the behavior of the Peltier element, a system-level model has been developed to simulate the behavior of the Peltier battery and predict the amount of heat it can transfer. The principle of

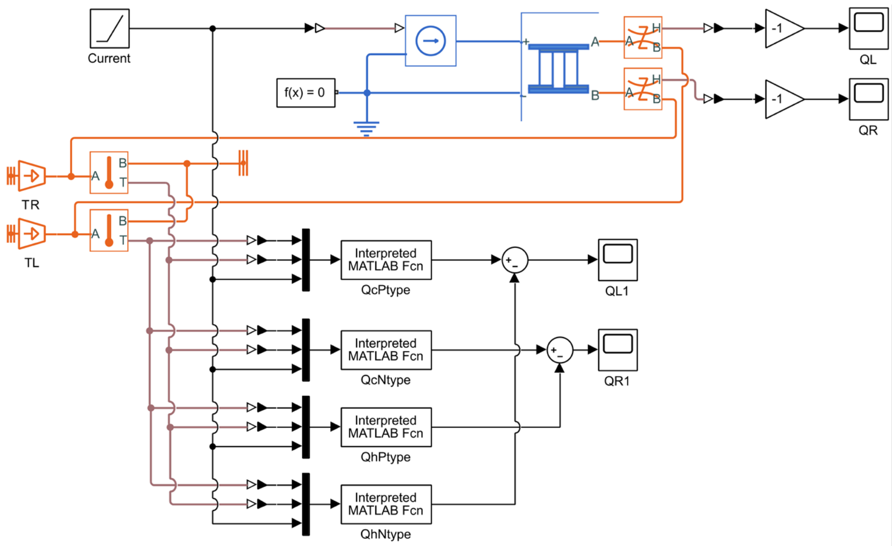

Fig. 6. Models of Peltier batteries

these models is to isolate the thermal and electrical parts of the system and to model them based on the thermal and electrical circuits with concentrated parameters [5, 6, 24]. Many works (see, for example, [5, 6]) are devoted to the study of the case of averaged material parameters, which do not depend on temperature and simplify the construction of equivalent circuits. Further, we propose to modify the scheme [5] shown in Fig. 6 by adding temperature-dependent material properties $\kappa = \kappa\left(u,\hat{T}\right)$, $S = S\left(u,\hat{T}\right)$, $\sigma = \sigma\left(u,\hat{T}\right)$, which are obtained after solving problems (4, 5).

The heat fluxes in the case of average material parameters Q_L and Q_R on the cold and hot sides of the Peltier element can be expressed as [10, 11]:

$$
\begin{aligned}
Q_L &= \frac{\left(T_L - T_R\right)}{\Theta} + SIT_L - \frac{1}{2}I^2R, \\
Q_R &= \frac{\left(T_L - T_R\right)}{\Theta} + SIT_R + \frac{1}{2}I^2R
\end{aligned}
\tag{19}
$$

and the voltage drop across the thermoelectric element is written as:

$$
V = S\left(T_R - T_L\right) + IR, \tag{20}
$$

where R is the electrical resistance, Θ is the thermal resistance, I is the electric current.

The averaged material parameters R, Θ, S in equations (19) are written in terms of n- and p-type material properties in the form [4]:

$$
\begin{aligned}
R &= \left(\frac{1}{<\sigma_n>} + \frac{1}{<\sigma_p>}\right)\frac{L}{A}, \quad S = \left|<S_n>\right| + \left|<S_p>\right|, \\
\Theta &= \left(\frac{1}{<\kappa_n>} + \frac{1}{<\kappa_p>}\right)\frac{L}{A}.
\end{aligned}
\tag{21}
$$

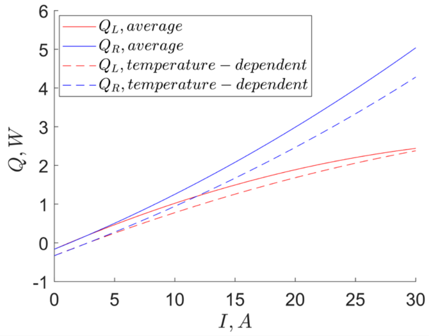

Fig. 7. Distribution of heat fluxes as a function of current strength
in the cases of asymptotic solution (dashed lines) and solution with averaged material parameters (solid line)

The Matlab/Simulink software package is used to build compact thermoelectric models of the Peltier element. In accordance with the above, Fig. 7 shows the diagrams of Peltier battery described by equations (16, 19) with material parameters given in Tables 1, 2. The upper part of the diagram shows the Peltier battery model with averaged material properties (21), which is specified by the "Peltier Device" block, and the lower part of the diagram shows the case of temperature-dependent material of the Peltier battery model (19). The defining equations of the thermal state of the "Peltier Device" block are similar to the expressions (19, 20). The lower blocks "QcPtype", "QcNtype", "QhPtype", "QcNtype" are a compact record of the heat flux expressions given by expressions (17).

Fig. 7 shows the dependences of heat fluxes Q_R, Q_L according to the defining equations (16, 19).

As can be seen from Fig. 7, the model of the Peltier element calculated with temperature-dependent parameters gives different values of heat fluxes in comparison with the model with averaged material parameters. This effect is because in the case of initial averaging of material parameters, the calculation of the thermal state is carried out at the temperature corresponding to the averaged values of material parameters. In the asymptotic solution, the system parameters depend on both the boundary conditions and the applied current, which leads to differences in the results.

Conclusion

In the present work, a model of reduced dimensionality for a Peltier element is constructed using the asymptotic direct expansion method. It is shown that taking into account temperature-dependent material parameters can significantly affect the boundary heat fluxes and integral characteristics of the thermoelectric device. The implementation of the proposed Peltier battery circuit in the Matlab/Simscape software package is demonstrated. The model can be useful for the analysis of quasi-stationary thermoelectric processes, when the total heat capacity of the Peltier battery is negligibly small in relation to the heat capacity of the thermostabilized object. The subject of further research may be the extension of the proposed approach to the modelling of fast non-stationary thermoelectric processes, in particular, to the modelling of the so-called "supercooling" phenomenon [11].

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