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ANALYSIS OF DIODE MIXERS USING NODAL VOLTAGE METHOD IN GENERALIZED MATRIX FORM IN FREQUENCY DOMAIN. PART 3: NONLINEAR DISTORTIONS

O.A. Golovan ✉

Peter the Great St. Petersburg Polytechnic University,
St. Petersburg, Russian Federation

✉ golovan.olga.andreevna@gmail.com

Abstract. A nonlinear distortions analysis method for diode frequency converter circuits is presented. Volterra series method is used to analyze nonlinear distortions. The analysis was carried out for three types of diode frequency converters: balanced, double balanced and triple balanced. Calculation of the 3rd order nonlinear distortion coefficient is presented. The dependences of the 3rd order nonlinear distortion coefficient on the load resistance and on the local oscillator (LO) voltage amplitude were obtained for two LO operation modes: harmonic and pulse. The error between the calculation and simulation results does not exceed 3 dB. It is shown that the dependences of the 3rd order nonlinear distortion coefficient on the load resistance and on the LO voltage amplitude have several maximums and minimums. By varying the values of the load resistance and the LO voltage amplitude it is possible to calculate the minimum achievable value of the 3rd order nonlinear distortion coefficient.

Keywords: diode frequency converters, nodal equations method, 3-rd order nonlinear distortions, Volterra series method, balanced mixer, double balanced mixer, triple balanced mixer

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АНАЛИЗ ДИОДНЫХ СМЕСИТЕЛЕЙ МЕТОДОМ УЗЛОВЫХ ПОТЕНЦИАЛОВ В ОБОБЩЕННОМ МАТРИЧНОМ ВИДЕ В ЧАСТОТНОЙ ОБЛАСТИ. ЧАСТЬ 3: НЕЛИНЕЙНЫЕ ИСКАЖЕНИЯ

О.А. Головань ✉

Санкт-Петербургский политехнический университет Петра Великого,
Санкт-Петербург, Российская Федерация

✉ golovan.olga.andreevna@gmail.com

Аннотация. Представлен метод анализа нелинейных искажений в схемах диодных преобразователей частоты. Для анализа нелинейных искажений использована методика рядов Вольтерра. Анализ проводился для трёх типов диодных преобразователей частоты – балансного, двойного балансного и тройного балансного. Представлен расчёт коэффициента нелинейных искажений по 3-й гармонике. Для двух режимов работы гетеродина – «неинтенсивного» и «интенсивного» – получены зависимости коэффициента нелинейных искажений по 3-й гармонике от сопротивления нагрузки и от амплитуды напряжения гетеродина. Ошибка между результатами расчёта и моделирования не превышает 3 дБ. Показано, что зависимости коэффициентов нелинейных искажений по 3-й гармонике от сопротивления нагрузки и от амплитуды напряжения гетеродина обладают несколькими максимумами и минимумами, что позволяет за счёт вариации значений сопротивления нагрузки и амплитуды напряжения гетеродина рассчитать минимально возможное значения коэффициента нелинейных искажений по 3-й гармонике.

Ключевые слова: диодные преобразователи частоты, метод узловых потенциалов, коэффициент нелинейных искажений по 3-й гармонике, метод рядов Вольтерра, балансные смесители, двойные балансные смесители, тройные балансные смесители

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Introduction

The paper is a sequel of [1, 2], devoted to the analysis of diode mixers using nodal equation method in frequency domain. As it was shown in the introduction of [1], the Volterra series method is widely used for nonlinear distortion analysis of parametric circuits. For example, in [3–6] nonlinear analysis of CMOS Gilbert cell mixer is presented, in particular, intermodulation distortions of the second and third orders are analyzed. In [7], the Volterra series method is applied to analyze the third-order nonlinear distortion of an anti-parallel diode pair mixers. Application of such a circuit allows the minimization of third-order intermodulation distortion. The dependence of the zero level of intermodulation distortion on the values of the source and load resistances is obtained. The choice of these resistances makes it possible to control the value of the intermodulation distortion minimum depending on the value of the bias voltage.

There are two sections in the present work. The first section presents a nonlinear analysis of diode mixers based on the Volterra series method. The decomposition of the diode current third harmonic in the frequency domain is obtained, the parametric model of the diode taking into account nonlinear effects is presented, and theoretical expressions for the 3rd order nonlinear distortion coefficients are found. The second section presents the calculation and simulation results for the 3rd order nonlinear distortion coefficients. Calculation and simulation were carried out for two LO operation modes: harmonic and pulse.

Nonlinear analysis of diode mixers using the Volterra series method

Representation of the diode current third harmonic

Assuming a large amplitude approximation of the LO. In this case the following relations are valid $U_{LOm} >> U_{0m}$, $U_{LOm} >> U_{IFm}$. The current through the nonlinear element of the mixer, i.e. the diode, is represented by a Taylor series, taking into account the terms up to third order

$$\begin{aligned} I \approx\ & f(U_{LO}) + \frac{\partial f(U_{LO})}{\partial U_0}U_0 + \frac{\partial f(U_{LO})}{\partial U_{IF}}U_{IF} + \frac{1}{2}\left[\frac{\partial^2 f(U_{LO})}{\partial U_0^2}U_0^2 + \right. \\ & \left. + 2\frac{\partial^2 f(U_{LO})}{\partial U_0 \partial U_{IF}}U_0 U_{IF} + \frac{\partial^2 f(U_{LO})}{\partial U_{IF}^2}U_{IF}^2\right] + \frac{1}{6}\left[\frac{\partial^3 f(U_{LO})}{\partial U_0^3}U_0^3 + \right. \\ & \left. + 3\frac{\partial^3 f(U_{LO})}{\partial U_0^2 \partial U_{IF}}U_0^2 U_{IF} + 3\frac{\partial^3 f(U_{LO})}{\partial U_0 \partial U_{IF}^2}U_0 U_{IF}^2 + \frac{\partial^3 f(U_{LO})}{\partial U_{IF}^3}U_{IF}^3\right] + \ldots = \\ & = I_0(U_{LO}) + G(U_{LO})U_0 + G(U_{LO})U_{IF} + \frac{1}{2}\left[G'(U_{LO})U_0^2 + \right. \\ & \left. + 2G'(U_{LO})U_0 U_{IF} + G'(U_{LO})U_{IF}^2\right] + \frac{1}{6}\left[G''(U_{LO})U_0^3 + 3G''(U_{LO})U_0^2 U_{IF} + \right. \\ & \left. + 3G''(U_{LO})U_0 U_{IF}^2 + G''(U_{LO})U_{IF}^3\right] + \ldots, \end{aligned} \tag{1}$$

where U_0 – is the input signal at the carrier frequency, U_{LO} – is the reference signal at the LO frequency, U_{IF} is the output signal at the intermediate frequency, $G(U_{LO}) = \frac{I_S}{\varphi_t}\left(e^{U_{LO}/\varphi_t}\right)$, $G'(U_{LO}) = \frac{1}{2}\frac{I_S}{\varphi_t^2}\left(e^{U_{LO}/\varphi_t}\right)$, $G''(U_{LO}) = \frac{1}{6}\frac{I_S}{\varphi_t^3}\left(e^{U_{LO}/\varphi_t}\right)$ are series coefficients, which are determined according to the Ebers-Moll model, φ_t is the thermopotential, I_S is the saturation current. Consider the harmonic LO operation mode, in which all signals (including the LO signal) are harmonic, and the initial phase, as in the case of linear analysis [1, 2], is assumed to be zero

$$U_0 = U_{0m}\cos\omega_0 t,\ \ U_{LO} = U_{LOm}\cos\omega_{LO} t,\ \ U_{IF} = U_{IFm}\cos\omega_{IF} t.$$

Let's represent the current expression (1) as

$$I = G(U_{LO})(U_0 + U_{IF}) + G'(U_{LO})(U_0 + U_{IF})^2 + G''(U_{LO})(U_0 + U_{IF})^3.$$

Assuming a balanced mixer structure, we neglect the second order effects, i.e. we consider only the third current harmonic

$$I_3 = G''(U_{LO})(U_0 + U_{IF})^3 = G''(U_{LO})\left(U_0^3 + 3U_0^2 U_{IF} + 3U_0 U_{IF}^2 + U_{IF}^3\right). \tag{2}$$

As the function $G(U_{LO})$ corresponds to the linear case [1, 2], the function $G''(U_{LO})$ is decomposed into a Fourier series of cosine harmonics with LO frequency ω_{LO}

$$G''(U_{LO}) = G_0'' + \sum_{n=1}^{\infty} G_n'' \cos n\omega_{LO} t \approx G_0'' + \sum_{n=1}^{N} G_n'' \cos n\omega_{LO} t,$$

where the coefficients G_n'' are related to the Fourier series coefficients g_n'' as $G_0'' = g_0''$, $G_n'' = 2g_n''$, $(n = 1,\ 2,\ \ldots\ N)$. For the harmonic LO operation mode, the expression for g_n'' are calculated as follows

$$g_n'' = \frac{1}{T}\int_0^T G''(U_{LO}) \cos n\omega_{LO} t dt = \frac{1}{6}\frac{I_S}{\varphi_t^3} B_n\left(U_{LOm}/\varphi_t\right),$$

where $T = 2\pi/\omega_{LO}$ is period of the LO frequency, $B_n\left(U_{LOm}/\varphi_t\right)$ is Bessel function of order n.

In pulse mode operation the Fourier series coefficients are calculated as

$$g_0'' = \frac{1}{T}\int_{-T/2}^{T/2} G''(U_{LO}) \cos(0\cdot\omega_{LO} t)\, dt = \frac{I_S}{12\varphi_t^3}\left(e^{\frac{U_{LOm}}{\varphi_t}} + e^{-\frac{U_{LOm}}{\varphi_t}}\right),$$

$$g_1'' = \frac{1}{T}\int_{-T/2}^{T/2} G''(U_{LO}) \cos \omega_{LO} t dt = \frac{I_S}{6\pi\varphi_t^3}\left(e^{\frac{U_{LOm}}{\varphi_t}} - e^{-\frac{U_{LOm}}{\varphi_t}}\right)\sin\frac{\pi}{2} = \frac{I_S}{6\pi\varphi_t^3}\left(e^{\frac{U_{LOm}}{\varphi_t}} - e^{-\frac{U_{LOm}}{\varphi_t}}\right).$$

Since second-order nonlinear effects for balance circuits are negligible, we take into account only the first LO harmonic in the series expansion of the parameter $G''(U_{LO})$

$$G''(U_{LO}) = G_0'' + G_1'' \cos \omega_{LO} t.$$

Then, according to formula (2), the third diode current harmonic is represented by

$$\begin{aligned} I_3(t) &= \left(G_0'' + G_1''\cos\omega_{LO} t\right)\left(U_0^3 + 3U_0^2 U_{IF} + 3U_0 U_{IF}^2 + U_{IF}^3\right) = \\ &= \left(G_0'' + G_1''\cos\omega_{LO} t\right)\Big(\left(U_{0m}\cos\omega_0 t\right)^3 + 3\left(U_{0m}\cos\omega_0 t\right)^2 U_{IFm}\cos\omega_{IF} t + \\ &\quad + 3U_{0m}\cos\omega_0 t\left(U_{IFm}\cos\omega_{IF} t\right)^2 + \left(U_{IFm}\cos\omega_{IF} t\right)^3\Big). \end{aligned}$$

After replacing the variable $\omega_{IF} = \omega_0 \pm \omega_{LO}$ and making some algebraic transformations, the expression for $I_3(t)$ is reduced to the form

$$\begin{aligned} I_3(t) &= \left(\frac{3G_0''U_{0m}U_{IFm}^2}{4} + \frac{3G_1''U_{0m}^2U_{IFm}}{8} + \frac{G_1''U_{IFm}^3}{8}\right)\cos(3\omega_0 \pm 2\omega_{LO})t + \\ &+ \left(\frac{3G_0''U_{0m}^2U_{IFm}}{4} + \frac{G_1''U_{0m}^3}{8} + \frac{3G_1''U_{0m}U_{IFm}^2}{8}\right)\cos(3\omega_0 \pm \omega_{LO})t + \\ &+ \left(\frac{3G_0''U_{IFm}^3}{4} + \frac{3G_1''U_{0m}U_{IFm}^2}{8}\right)\cos(3\omega_0 \pm 3\omega_{LO})t + \frac{G_1''U_{IFm}^3}{8}\cos(3\omega_0 \pm 4\omega_{LO})t + \\ &+ \left(\frac{9G_0''U_{0m}^2U_{IFm}}{4} + \frac{3G_0''U_{IFm}^3}{4} + \frac{3G_1''U_{0m}^3}{8} + \frac{9G_1''U_{0m}U_{IFm}^2}{8}\right)\cos(\omega_0 \pm \omega_{LO})t + \\ &+ \left(\frac{3G_0''U_{0m}U_{IFm}^2}{4} + \frac{9G_1''U_{0m}^2U_{IFm}}{8} + \frac{3G_1''U_{IFm}^3}{8}\right)\cos(\omega_0 \pm 2\omega_{LO})t + \end{aligned}$$

$$+\frac{3G_1''U_{0m}U_{IFm}^2}{8}\cos\left(\omega_0 \pm 3\omega_{LO}\right)t+\left(\frac{G_0''U_{0m}^3}{4}+\frac{3G_1''U_{0m}^2U_{IFm}}{4}\right)\cos 3\omega_0 t+$$
$$+\left(\frac{G_0''U_{0m}^3}{4}+\frac{G_0''U_{0m}U_{IFm}^2}{2}+\frac{3G_1''U_{0m}^2U_{IFm}}{4}+\frac{G_1''U_{IFm}^3}{4}\right)\cos\omega_0 t.$$

The sum of terms in the expression for $I_3\left(t\right)$ corresponds to the combinational harmonics generated by mixer. To analyze the 3rd order nonlinear distortion we consider only the terms $\cos\left(3\omega_0 \pm 3\omega_{LO}\right)t$, $\cos\left(\omega_0 \pm \omega_{LO}\right)t,\ \cos 3\omega_0 t,\ \cos\omega_0 t$

$$I_3\left(t\right)=\left(\frac{9G_0''U_{0m}^2U_{IFm}}{4}+\frac{3G_0''U_{IFm}^3}{4}+\frac{3G_1''U_{0m}^3}{8}+\frac{9G_1''U_{0m}U_{IFm}^2}{8}\right)\cos\left(\omega_0 \pm \omega_{LO}\right)t+$$
$$+\left(\frac{3G_0''U_{IFm}^3}{4}+\frac{3G_1''U_{0m}U_{IFm}^2}{8}\right)\cos\left(3\omega_0 \pm 3\omega_{LO}\right)t+\left(\frac{G_0''U_{0m}^3}{4}+\frac{3G_1''U_{0m}^2U_{IFm}}{4}\right)\cos 3\omega_0 t+$$
$$+\left(\frac{G_0''U_{0m}^3}{4}+\frac{G_0''U_{0m}U_{IFm}^2}{2}+\frac{3G_1''U_{0m}^2U_{IFm}}{4}+\frac{G_1''U_{IFm}^3}{4}\right)\cos\omega_0 t.$$

Let us rewrite this expression by replacing the cosine of the triple angle by the cube of the cosine

$$I_3\left(t\right)=\left(G_0''U_{IFm}^3+\frac{3}{2}G_1''U_{0m}U_{IFm}^2\right)\cos^3\left(\omega_0 \pm \omega_{LO}\right)t+\left(G_0''U_{0m}^3+3G_1''U_{0m}^2U_{IFm}\right)\cos^3\omega_0 t+$$
$$+\left(\frac{3}{2}G_0''U_{0m}U_{IFm}^2+\frac{3}{4}G_1''U_{IFm}^3\right)\cos\omega_0 t+\left(\frac{9}{4}G_0''U_{0m}^2U_{IFm}+\frac{3}{8}G_1''U_{0m}^3\right)\cos\left(\omega_0 \pm \omega_{LO}\right)t.$$

The amplitude values of the input U_{0m} and output U_{IFm} signals are related by the mixer conversion gain K as $U_{IFm}=KU_{0m}$. Then, the expression for the current third harmonic is as follows

$$I_3\left(t\right)=\left(G_0''U_{IFm}^3+\frac{3}{2}G_1''U_{0m}U_{IFm}^2\right)\cos^3\left(\omega_0 \pm \omega_{LO}\right)t+\left(G_0''U_{0m}^3+3G_1''U_{0m}^2U_{IFm}\right)\cos^3\omega_0 t+$$
$$+\left(\frac{3}{2}G_0''U_{0m}\left(KU_{0m}\right)^2+\frac{3}{4}G_1''\left(KU_{0m}\right)^2U_{IFm}\right)\cos\omega_0 t+$$
$$+\frac{9}{4}\left(G_0''U_{0m}^2U_{IFm}+\frac{3}{8}G_1''U_{0m}^3\right)\cos\left(\omega_0 \pm \omega_{LO}\right)t.$$

The transformation from time domain to frequency domain is performed using the three-dimensional Laplace transform on the argument $j\omega_0$. In this case, the diode current third harmonic in the frequency domain is represented as

$$I_3\left(p,p,p\right)=G_0''\left(U_{IF}\left(p_0 \pm j\omega_{LO}\right)\right)^3+\frac{3}{2}G_0''\left(KU_{0m}\right)^2U_0\left(p_0\right)+$$
$$+\frac{3}{2}G_1''U_0\left(p_0 \pm j\omega_{LO}\right)\left(U_{IF}\left(p_0 \pm j\omega_{LO}\right)\right)^2+G_0''\left(U_0\left(p_0\right)\right)^3+3G_1''\left(U_0\left(p_0\right)\right)^2U_{IF}\left(p_0\right)+$$
$$+\frac{3}{4}G_1''\left(KU_{0m}\right)^2U_{IF}\left(p_0\right)+\frac{9}{4}G_0''U_{0m}^2U_{IF}\left(p_0 \pm j\omega_{LO}\right)+\frac{3}{8}G_1''U_{0m}^2U_0\left(p_0 \pm j\omega_{LO}\right).$$

Nonlinear distortion analysis in the balanced diode mixer

The schematic and equivalent circuits of the balance mixer are presented in [1, Fig. 3]. To analyze the 3rd order nonlinear distortion, an equivalent circuit (Fig. 1) is designed, the feature of which, compared to the linear analysis, is the replacement of voltage arguments in the expressions for the current generators

$$0,5G_{1i}U_{0i}\left(p_0 \pm j\omega_{LO},\ p_0 \pm j\omega_{LO}\ \ p_0 \pm j\omega_{LO}\right),\ \ G_{1i}U_{IFi}\left(p_0, p_0, p_0\right).$$

For each diode, two additional current generators C_i and D_i (index i = 1, 2 corresponds to the diode number) are introduced. These generators describe the effect of the current 3rd harmonic $I_{3i}\left(p,p,p\right)=$ $=C_i+D_i$. The expressions for generators C_i include terms corresponding to the effect of the signal at the carrier frequency, and the expressions for generators D_i include terms corresponding to the effect of the signal at the intermediate frequency

$$C_i=\frac{3}{2}G''_{0i}\left(KU_{0im}\right)^2U_{0i}\left(p_0\right)+\frac{3}{8}G''_{1i}U_{0im}^2U_{0i}\left(p_0\pm j\omega_{LO}\right)+G''_{0i}\left(U_{0i}\left(p_0\right)\right)^3+$$
$$+\frac{3}{2}G''_{0i}U_{0i}\left(p_0\pm j\omega_{LO}\right)\left(U_{IFi}\left(p_0\pm j\omega_{LO}\right)\right)^2,$$
$$D_i=G''_{0i}\left(U_{IFi}\left(p_0\pm j\omega_{LO}\right)\right)^3+\frac{9}{4}G''_{0i}U_{0im}^2U_{IFi}\left(p_0\pm j\omega_{LO}\right)+$$
$$+3G''_{1i}\left(U_{0i}\left(p_0\right)\right)^2U_{IFi}\left(p_0\right)+\frac{3}{4}G''_{1i}\left(KU_{0im}\right)^2U_{IFi}\left(p_0\right).$$

The equations C_i and D_i include the voltages $U_{0i}\left(p_0\right)$, $U_{0i}\left(p_0\pm j\omega_{LO}\right)$, $U_{IFi}\left(p_0\pm j\omega_{LO}\right)$, and $U_{IFi}\left(p_0\right)$, whose expressions are determined during the linear analysis (see [1], Section 2). The circuit in Fig. 1 is described by two coupled systems of nodal equations. The system of nodal equations on the arguments $\left(p_0, p_0, p_0\right)$ in matrix form is

$$\left[Y\left(p_0+p_0+p_0\right)\right]\begin{bmatrix}U_1\left(p_0,p_0,p_0\right)\\U_2\left(p_0,p_0,p_0\right)\\U_3\left(p_0,p_0,p_0\right)\\U_4\left(p_0,p_0,p_0\right)\\U_5\left(p_0,p_0,p_0\right)\\U_6\left(p_0,p_0,p_0\right)\end{bmatrix}=\begin{bmatrix}0\\0\\0,5G_{11}U_{01}\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)+C_1\\0,5G_{12}U_{02}\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)+C_2\\-0,5G_{11}U_{01}\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)-C_1\\-0,5G_{12}U_{02}\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)-C_2\end{bmatrix}. \qquad (3)$$

The system of nodal equations on the arguments $\left(p_0\pm j\omega_{LO},\ p_0\pm j\omega_{LO},\ p_0\pm j\omega_{LO}\right)$ in matrix form is

$$\left[Y\left(p_0\pm j\omega_{LO}+p_0\pm j\omega_{LO}+p_0\pm j\omega_{LO}\right)\right]\times$$
$$\times\begin{bmatrix}U_1\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\\U_2\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\\U_3\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\\U_4\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\\U_5\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\\U_6\left(p_0\pm j\omega_{LO},p_0\pm j\omega_{LO},p_0\pm j\omega_{LO}\right)\end{bmatrix}=\begin{bmatrix}0\\0\\G_{11}U_{IF1}\left(p_0,p_0,p_0\right)+D_1\\G_{12}U_{IF2}\left(p_0,p_0,p_0\right)+D_2\\-G_{11}U_{IF1}\left(p_0,p_0,p_0\right)-D_1\\-G_{12}U_{IF2}\left(p_0,p_0,p_0\right)-D_2\end{bmatrix}. \qquad (4)$$

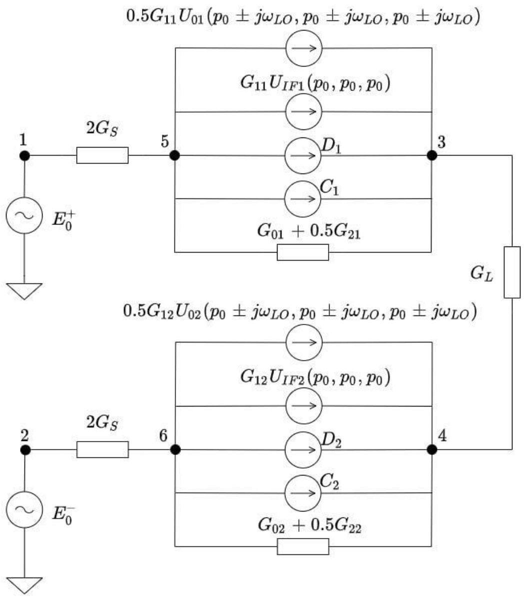

Fig. 1. Equivalent circuit of a balanced diode mixer for analyzing the coefficient of nonlinear distortion in the 3rd harmonic

If there are no reactance circuit elements the Y-matrix is equal to

$$\left[Y\left(p_0 \pm j\omega_{LO} + p_0 \pm j\omega_{LO} + p_0 \pm j\omega_{LO}\right)\right] = \left[Y\left(p_0 + p_0 + p_0\right)\right] = \left[Y\right],$$

where

$$\left[Y\right] = \begin{bmatrix} G+2G_S & 0 & 0 & 0 & -2G_S & 0 \\ 0 & G+2G_S & 0 & 0 & 0 & -2G_S \\ 0 & 0 & G_{d1}+G_L & -G_L & -G_{d1} & 0 \\ 0 & 0 & -G_L & G_{d2}+G_L & 0 & -G_{d2} \\ -2G_S & 0 & -G_{d1} & 0 & G_{d1}+2G_S & 0 \\ 0 & -2G_S & 0 & -G_{d2} & 0 & G_{d2}+2G_S \end{bmatrix},$$

where for reducing the recording, we introduced the notations $G_{0i} + 0,5G_{2i} = G_{di},\ i = 1, 2.$ Conductance G is introduced into the circuit to convert the input voltage source to a current source. The voltages $U_{0i}\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right)$ and $U_{IFi}\left(p_0, p_0, p_0\right)$ are expressed through the nodal potential as

$$U_{01}\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right) = U_5\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right) - $$
$$- U_3\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right),$$
$$U_{02}\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right) = U_6\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right) - $$
$$- U_4\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right),$$
$$U_{IF1}\left(p_0, p_0, p_0\right) = U_5\left(p_0, p_0, p_0\right) - U_3\left(p_0, p_0, p_0\right),$$
$$U_{IF2}\left(p_0, p_0, p_0\right) = U_6\left(p_0, p_0, p_0\right) - U_4\left(p_0, p_0, p_0\right).$$

Using the approach to the solution of systems (3) and (4) presented in [1], we obtain expressions for the vectors of nodal potentials. The coefficient of the 3rd order nonlinear distortion is equal to

$$K_3 = \lim_{G\to\infty} \frac{U_{out}\left(p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}, p_0 \pm j\omega_{LO}\right)}{U_{out}\left(p_0 \pm j\omega_{LO}\right)U_{out}\left(p_0 \pm j\omega_{LO}\right)U_{out}\left(p_0 \pm j\omega_{LO}\right)},$$

where the voltage $U_{out}\left(p_0 \pm j\omega_{LO}\right)$ is obtained during the linear analysis. For the same diode parameters $C_1 = C_2 = C$ and $D_1 = D_2 = D$ the coefficient of the 3rd order nonlinear distortion is equal to

$$K_3 = \Big\{\Big[CG_1\left(G_S + G_L\right) - D\Big(2G_SG_L + \left(G_0 + 0{,}5G_2\right)G_L + G_S\left(G_0 + 0{,}5G_2\right)\Big)\Big]\times \\ \Big[\Big(2\big(\left(G_0 + 0{,}5G_2\right)\left(G_S + G_L\right) + 2G_SG_L\big)^2 - G_1^2\left(G_S + G_L\right)^2\Big)^2\Big]\Big\} \Big/ \Big\{8E_0^3G_L^3G_1^3G_S^5\Big\}, \tag{5}$$

where E_0 is the input generator ([1], Fig. 3).

Nonlinear distortion analysis in the double balanced mixer

The schematic and equivalent circuits of the double balanced diode mixer are introduced in [1, Fig. 5]. The double balanced mixer circuit is represented as a parallel connection of two balanced mixers. Consequently, the 3rd order nonlinear distortion coefficient of the double balanced circuit is defined as the sum of the 3rd order nonlinear distortion coefficients of each diode pair. The first diode pair is a balanced diode mixer, the expression for the 3rd order nonlinear distortion coefficient of which K_{31} corresponds to formula (5). The 3rd order nonlinear distortion coefficient for the second diode pair K_{32} also corresponds to formula (5). Thus, the 3rd order nonlinear distortion coefficient of the double balanced mixer finally is given as

$$K_3 = K_{31} + K_{32} = \Big\{\Big[CG_1\left(G_S + G_L\right) - D\Big(2G_SG_L + \left(G_0 + 0{,}5G_2\right)G_L + G_S\left(G_0 + 0{,}5G_2\right)\Big)\Big]\times \\ \Big[\Big(2\big(\left(G_0 + 0{,}5G_2\right)\left(G_S + G_L\right) + 2G_SG_L\big)^2 - G_1^2\left(G_S + G_L\right)^2\Big)^2\Big]\Big\} \Big/ \Big\{4E_0^3G_L^3G_1^3G_S^5\Big\}.$$

Nonlinear distortion analysis in the triple balanced mixer

The schematic and equivalent circuits of the triple balanced diode mixer are introduced in [1, Fig. 7]. The triple balanced mixer circuit is represented as a parallel connection of two double balanced mixers or as a parallel connection of four balanced mixers. Consequently, the 3rd order nonlinear distortion coefficient of a triple balanced circuit is defined as the sum of the 3rd order nonlinear distortion coefficients of the four balanced mixers or two double balanced mixers. The 3rd order nonlinear distortion coefficient of a triple balanced mixer is

$$K_3 = K_{31} + K_{32} + K_{33} + K_{34} = \\ = \Big\{\Big[CG_1\left(G_S + G_L\right) - D\Big(2G_SG_L + \left(G_0 + 0{,}5G_2\right)G_L + G_S\left(G_0 + 0{,}5G_2\right)\Big)\Big]\times \\ \Big[\Big(2\big(\left(G_0 + 0{,}5G_2\right)\left(G_S + G_L\right) + 2G_SG_L\big)^2 - G_1^2\left(G_S + G_L\right)^2\Big)^2\Big]\Big\} \Big/ \Big\{2E_0^3G_L^3G_1^3G_S^5\Big\}.$$

Calculation and simulation

In the calculation and simulation of all mixer circuits have been used frequency ranges, the values of the diode model parameters and circuit elements are the same as in [1, 2]: the diode saturation current I_S = 1.14 pA, the input signal voltage amplitude U_{0m} = 0.05 V, the LO voltage amplitude U_{LOm} = 1.0 V,

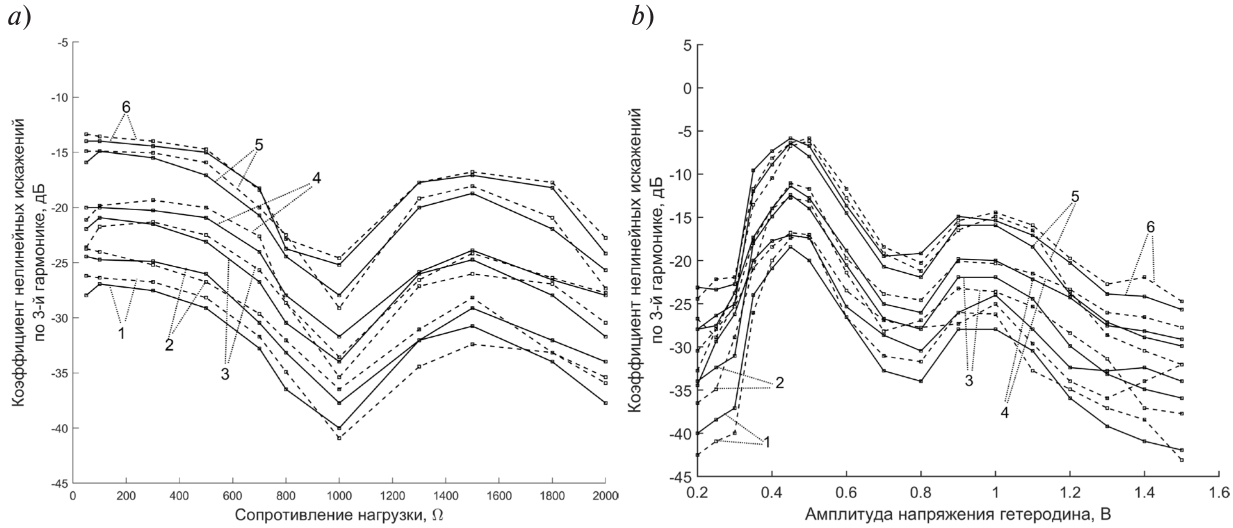

Fig. 2. Dependence of the coefficients of nonlinear distortion in the 3-rd harmonic on the load resistance (*a*), on the amplitude of the voltage of the heterodyne (b). Solid line – calculation, dotted line – modeling. "Non–intensive" mode of operation of the heterodyne: balanced circuit – 1, double balanced – 3, triple balanced – 5; "intensive" mode of operation of the heterodyne: balanced circuit – 2, double balanced – 4, triple balanced – 6

Table 1

Results of calculation and modeling of the nonlinear distortion coefficient

Type of scheme	The mode of operation of the heterodyne	Coefficient of nonlinear distortion of the 3rd harmonic, dB	
		Calculation	Modeling
B	"Non–intensive"	–27.7	–26.2
	"Intensive"	–24.0	–25.0
DB	"Non–intensive"	–21.7	–20.0
	"Intensive"	–20.0	–20.4
TB	"Non–intensive"	–15.7	–14.5
	"Intensive"	–15.4	–14.1

the source resistance $R_S = 50\ \Omega$, the load resistance $R_L = 50\ \Omega$; the input signal frequency is 4 MHz, the LO frequency is 5 MHz. The results of calculation and simulation for two LO operation modes (harmonic and pulse) are given in Table 1.

Fig. 2 a shows the dependences of the 3rd order nonlinear distortion coefficients on the load resistance at $U_{LOm} = 1$ V for two LO operation modes, and Fig. 2 b shows the dependences of the 3rd order nonlinear distortion coefficients on the LO voltage amplitude at $R_L = 50\ \Omega$ for two LO operation modes. The results of simulation confirm the calculation accuracy, the error does not exceed 3 dB.

Conclusion

The analysis method of nonlinear distortion in diode mixers using Volterra series is presented. The values of the 3rd order nonlinear distortion coefficients for the LO harmonic operation mode were calculated (and simulated): –27.7 dB (–26.2 dB) for balanced circuit, –21.7 dB (–20.0 dB) for double balanced circuit, –15.7 dB (–14.5 dB) for triple balanced circuit. In addition, the values of the 3rd order nonlinear distortion coefficients for the LO pulse operation mode were calculated (and simulated): −24.0 dB (−25.0 dB) for balanced circuit, −20.0 dB (−20.4 dB) for double balanced circuit, −15.4 dB (−14.1 dB) for triple balanced circuit. The dependences of the 3rd order nonlinear distortion coefficient on the load resistance and on the LO voltage amplitude are obtained. The error between the calculation and simulation results does not exceed 3 dB. It is shown that the dependences of the 3rd order nonlinear distortion coefficient on the load resistance and on the LO voltage amplitude have several maximums and minimums. Thus, by varying the values of R_L and U_{LOm} it becomes possible to calculate the minimum achievable value of the 3rd order nonlinear distortion coefficient. For example, in the harmonic LO operation mode at LO voltage amplitude of 1 V at a load resistance of 1000 Ω the minimum of the 3rd order nonlinear distortion coefficient is −40.0 dB; at load resistance of 50 Ω the minimum of the 3rd order nonlinear distortion coefficient is achieved at LO voltage amplitude of 0.8 V and corresponds to −34.0 dB. The nonlinear analysis of diode mixers showed that the balanced mixer has the lowest 3rd order nonlinear distortion coefficient, and the triple balanced mixer has the highest one, the LO harmonic operation mode provides lower values of nonlinear distortion coefficients than the pulse operation mode. The nonlinear distortion analysis method presented in this paper allows estimation of the nonlinear distortion level caused not only by the third harmonic, but also by harmonics of higher orders.

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INFORMATION ABOUT AUTHOR / СВЕДЕНИЯ ОБ АВТОРЕ

Golovan Olga A.
Головань Ольга Андреевна
E-mail: golovan.olga.andreevna@gmail.com
ORCID: https://orcid.org/0000-0002-0950-1333

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