Computing, Telecommunication and Control, 2023, Vol. 16, No. 2, Pp. 40–54.
Информатика, телекоммуникации и управление. 2023. Том 16, № 2. С. 40–54.

Research article
DOI: https://doi.org/10.18721/JCSTCS.16204
UDC 621.396.96

MULTICHANNEL MULTISTATIC COMBINED TSOA AND TDOA POSITIONING SYSTEM BASED ON PRECISE ANALYTICAL SOLUTION OF POSITIONING EQUATIONS

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Abstract. Exact analytical solutions of elliptic and hyperbolic equations in a multichannel multistatic positioning system are obtained. The results of modeling the Time Sum of Arrival (TSoA) and Time Difference of Arrival (TDoA) methods for estimating the object coordinates by arrival times with fluctuations are presented. Particular attention is paid to the causes of gross errors in case of problematic configurations of base stations (BS) and the position of the object. The study revealed that the advantages of the TSoA method over the TDoA method include a reduction in the area with anomalous errors and better accuracy outside the BS perimeter, and the advantage of TDoA over TSoA should include more accurate work inside the BS perimeter. Based on the identified advantages, optimization of a multichannel multistatic positioning system, which combines the advantages of TSoA and TDoA methods, is proposed. As a result of the simulation, it was found that the combined TSoA/TDoA method, based on the exact analytical solution of equations, has an order of magnitude higher accuracy in determining the object's coordinates than the frequently used method of linearization of hyperbolic equations. Due to these advantages, the proposed algorithm is promising for remote determination of the parameters of unmanned vehicles in "smart city" technologies.

Keywords: positioning; Time Difference of Arrival; Time Sum of Arrival; desynchronization in time of arrival; optimization of a positioning system; coordinate estimation; combined TSoA/TDoA

Acknowledgements: The reported study was funded by RFBR, project number 19-29-06034.

Citation: Kuptsov V.D., Ivanov S.I. Multichannel multistatic combined TSoA and TDoA positioning system based on precise analytical solution of positioning equations. Computing, Telecommunications and Control, 2023, Vol. 16, No. 2, Pp. 40–54. DOI: 10.18721/JCSTCS.16204

Научная статья
DOI: https://doi.org/10.18721/JCSTCS.16204
УДК 621.396.96

МНОГОКАНАЛЬНАЯ МУЛЬТИСТАТИЧЕСКАЯ КОМБИНИРОВАННАЯ СИСТЕМА ПОЗИЦИОНИРОВАНИЯ TSOA И TDOA, ОСНОВАННАЯ НА ТОЧНОМ АНАЛИТИЧЕСКОМ РЕШЕНИИ УРАВНЕНИЙ ПОЗИЦИОНИРОВАНИЯ

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Аннотация. Получены точные аналитические решения эллиптических и гиперболических уравнений в многоканальной мультистатической системе позиционирования. Приведены результаты моделирования суммарно-дальномерного (TSoA) и разностно-дальномерного (TDoA) методов для оценки координат объекта по времени прихода с флуктуациями. Особое внимание уделено причинам возникновения грубых ошибок при проблемных конфигурациях базовых станций (БС) и положения объекта. В ходе исследования выявлено, что преимущества метода TSoA перед методом TDoA заключаются в уменьшении площади с аномальными ошибками и лучшей точности за пределами периметра БС, а преимущество TDoA перед методом TSoA заключается в более точной работе внутри периметра БС. На основе выявленных преимуществ предлагается оптимизация многоканальной мультистатической системы позиционирования, которая сочетает в себе преимущества методов TSoA и TDoA. В результате моделирования установлено, что комбинированный метод TSoA/TDoA, основанный на точном аналитическом решении уравнений, имеет на порядок более высокую точность определения координат объекта, чем часто используемый метод линеаризации гиперболических уравнений. Благодаря этим достоинствам предложенный алгоритм перспективен для дистанционного определения параметров беспилотных автомобилей в технологиях «умного города».

Ключевые слова: позиционирование; разностно-дальномерный метод; суммарно-дальномерный метод; рассинхронизация во времени прихода; оптимизация системы позиционирования; оценка координат; комбинированный TSoA/TDoA

Благодарности: Исследование выполнено при финансовой поддержке РФФИ, проект № 19-29-06034.

Для цитирования: Kuptsov V.D., Ivanov S.I. Multichannel multistatic combined TSoA and TDoA positioning system based on precise analytical solution of positioning equations // Computing, Telecommunications and Control. 2023. Т. 16, № 2. С. 40–54. DOI: 10.18721/JCSTCS.16204

Introduction

Accurate remote determination of the object coordinates on the plane and in space is necessary in many technical applications, such as the control of unmanned vehicles in the "smart city", in robotic production complexes, security systems in the city and at the workplace. The object coordinates are determined by several base stations (BS) receiving a signal from the object. In global positioning systems, for example GPS, the radio signal is received by navigation satellites; in local positioning systems, object signals are received by base stations. The main task of object positioning is to determine the object coordinates on the ground, which can be carried out using various algorithms. The main ones are: RSS (Received Strength Signal), AoA (Angle of Arrival), ToA (Time of Arrival), TDoA (Time Difference of Arrival) and TSoA (Time Sum of Arrival) [1].

Each of the methods has its own disadvantages and advantages. In the RSS method, the distance to an object is estimated by the power of the radio signal received at the base stations and emitted by the object. Hardware implementation of RSS measurements is relatively inexpensive due to the simplicity of the measurement data processing algorithm. However, the results of measurements of the power of the received radio signal for the RSSI algorithm are difficult to describe by theoretical models, and, as a result, are poorly predictable. The RSS method loses a lot in conditions of multipath signal propagation. In the AoA method, the location of an object is determined by the intersection of the axes of the antenna patterns of three base stations (modified triangulation method). The method has a great advantage – there is no need for precise time synchronization of transmitters and receivers. The disadvantage is that at base stations, receivers must have a very high angular resolution, which requires the use of multi-element phased antenna arrays. In the range of units of gigahertz, such receivers have significant dimensions, with an increase in radio frequency, the dimensions decrease, but the absorption of radio waves during propagation increases [2]. In the ToA method, the signal transit times from the object to the base stations are measured, the distance to the object is calculated based on the difference in the time of sending and receiving the signal. The disadvantage of the method is that it requires complete time synchronization of the transmitter at the object and the receivers at the base stations.

In the complex changing conditions of a "smart city" and a production facility, the TDoA method is preferred, since this method is highly accurate and requires time synchronization only at base stations [3]. In the TDoA method, the differences in the arrival times of radio signals between the reference BS and all other BSs are measured. The lines on which the difference in the times of arrival of radio signals to two BSs is constant are hyperbolas in the plane (hyperboloids in space), while both BSs are at the foci of the hyperbolas. The object's coordinates are the point of intersection of the entire set of hyperbolic curves in the plane, included in the system of equations. To obtain a position estimate at reasonable noise levels, the Taylor series method [4] is also used. In [5], it was proposed to use the expansion in a Taylor series and the linearization of hyperbolic equations. Certain advantages are provided by combined methods, for example, TDoA/RSS [6]. The use of TDoA/RSS reduces the number of required BSs to two. Good results in determining the position of an object are obtained by synthetic aperture radars using the time difference of arrival (TDoA) method [7].

The option of constructing a positioning system on a plane with 3 BS is the most cost-effective, but has the disadvantage that the intersection of two hyperbolas can occur at four points. In this case, it becomes problematic to identify the true position of the object. To eliminate the ambiguity in determining the position of an object, it is necessary to include a fourth BS in the positioning system. With the addition of the fourth BS, it becomes possible to solve three systems of equations that will have one common root corresponding to the true position of the target [13]. To solve the problem of positioning in space, the number of BS should be equal to five.

Of particular interest are studies in which the hyperbolic equations of the TDoA method are solved analytically, without the linearization approximation. This solution improves the accuracy and reliability of determining the coordinates of the object. A certain disadvantage of the analytical method for solving equations is the increased requirement for the processing power of the positioning system processor, since it becomes necessary to solve quadratic equations. However, with the development of microprocessor technology, this disadvantage is leveled. Analytical methods for solving hyperbolic equations in the TDoA method are presented in [9–11]. Transformation of the coordinates of a hyperbola during a shift and rotation on a plane can be found in [9]. According to this transformation in [10], the effectiveness of two methods of TDOA solution was compared based on the task of finding intersection points of hyperboloids (possible positions of a target). The first method analyzed was based on coordinate transformation from the initial system to a new system to simplify equations solving, and the second one was based on matrix. In [11], the results of an experimental verification of an analytical method based on coordinate transformation are presented. In [12] an analytical method for determining the position of an object based on the analysis of the difference in time of arrival of microwave radar signals from a transmitter to base station receivers is proposed. The main feature of the method is that it allows you to eliminate the ambiguity in determining the 3D coordinates of the target and improve the accuracy of determining coordinates in the absence of synchronization of TDoA measurements between the object transmitter and BS receivers.

Algorithms based on TSoA are less frequently studied than TDoA algorithms, and the available articles note certain advantages of the TSoA method over the TDoA method [13]. Minor disadvantages of the TSoA method include the need for a larger number of ADC bits during analog-to-digital conversion since the sum signal has a larger value than the difference signal. In [13] investigates the time-sum-of-arrival (TSoA) based localization algorithm, where a two-step weighted least-squares algorithm is analytically derived according to the elliptical geometry. Simulations conducted in a wide range of scenarios show that in the presence of a large number of BSs, the TSoA-based algorithm performs better than the TDoA-based algorithm. In the scenario where the target is located near the cell boundary, the TSoA-based algorithm has better accuracy performance compared to the TDoA-based algorithm. In [14], the problem of two-dimensional elliptic localization with several spaced transmitters and receivers is considered. To determine the location, the signal reflected from the object (or relayed) is used under NLOS conditions of radio signal propagation. Two least squares methods are investigated: the use of the traditional linear LS estimate with an additional cost function; and parametrizing the ellipses defined by TSoA by the non-linear LS estimation criterion. The effectiveness of the proposed methods in reducing NLOS errors at acceptable computational costs is demonstrated. A novel computationally efficient solution for locating a single target from bistatic range measurements in distributed MIMO radar systems is proposed in [15]. The method is based on the method of least squares with restrictions. The positioning performance of the proposed method is shown to achieve the CRLB up to relatively high noise levels. A statistical model of the NLOS error in positioning systems based on the trilateration method was proposed in [16]. The maximum likelihood estimate (MLE) of the object location is calculated. With the help of Gaussian approximation, a unified (taking into account the measurements of ToA, TSoA and TDoA) closed form expression for the CRLB of the estimated object location variance is obtained. In [17] the problem of multistatic target localization using TSoA measurements in the presence of non-line-of-sight (NLOS) propagation errors and bursts in a MIMO radar system consisting of a single target, M transmitters, and N receivers is considered. The influence of NLOS errors or any other outliers in the measurements is eliminated by introducing an additional balancing parameter into the data model, after which the iterative algorithm for minimizing the NLWLS objective function is used. The TSoA-based Taylor series localization algorithm has been deeply investigated in [18], where the principle and motivation of this algorithm has been introduced. Simulations show that a large number of BSs will make the TSOA-based Taylor algorithm perform better than the TDOA based algorithm. Moreover, when object is far from reference BS, TSOA-based Taylor algorithm is also better than the TDOA based algorithm. In [19] derives a new algebraic positioning solution using a minimum number of measurements, and from which to develop an outlier detector and an object location estimator. It is claimed that two measurements are sufficient in 2-D and three in 3-D to yield a solution if they are consistent. The derived minimum measurement solution is exact and reduces the computation to the roots of a quadratic equation. The solution derivation leads to simple criteria to ascertain if the line of positions from two measurements intersect. The intersection condition enables us to establish an outlier detector based on graph processing.

The aim of this work is to develop the combined TSoA/TDoA algorithm to improve the accuracy of determining the coordinates of an object and reduce gross errors associated with zeroing the determinants of equations systems. To achieve that, elliptic and hyperbolic positioning equations were solved, the standard deviation of the object coordinates was simulated with fluctuations in arrival times by linearization methods and the exact analytical solution of hyperbolic and elliptic equations, were identified areas on the plane in which gross errors associated with the zeroing of the determinants of equations systems occur.

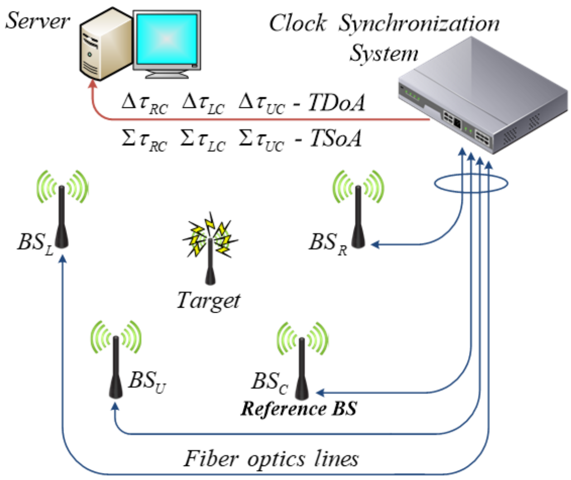

Fig. 1. Block diagram of a positioning system with 4 BS

TSoA and TDoA positioning systems

Let consider a positioning system on a plane including four radio signal receivers: BS_C, BS_L, BS_R and BS_U (Fig. 1).

In the absence of TDoA measurement errors and receiving line-of-sight signals the real values of the sum and difference of the signal arrival times between the BSi and the reference BS_C are determined by the expression:

$$\Delta\tau_{iC} = \frac{r_i \pm r_C}{c_l} = \frac{r_{iC}}{c_l}, \quad i = 2, \ldots 3, \tag{1}$$

where c_l – speed of light, $r_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$ – distance between object and BSi, r_C – distance between the object and BS_C, $[x_i, y_i]$ – BSi coordinates, $[x, y]$ – object coordinates.

The transmitter position coordinates are solutions of the system of elliptic (TSoA method) and hyperbolic (TDoA method) equations:

$$\begin{cases} \sqrt{(x-x_L)^2 + (y-y_L)^2} \pm \sqrt{(x-x_C)^2 + (y-y_C)^2} = c \cdot \Delta\tau_{LC} \\ \sqrt{(x-x_R)^2 + (y-y_R)^2} \pm \sqrt{(x-x_C)^2 + (y-y_C)^2} = c \cdot \Delta\tau_{RC} \\ \sqrt{(x-x_U)^2 + (y-y_U)^2} \pm \sqrt{(x-x_C)^2 + (y-y_C)^2} = c \cdot \Delta\tau_{UC} \end{cases}, \tag{2}$$

where $\Delta\tau_{LC}$, $\Delta\tau_{RC}$, $\Delta\tau_{UC}$ – the sum or difference in the times of arrival of the radio signal between $BS_{L,\ R,\ U}$ and the BSC reference station. The elliptic equations of the TSoA method correspond to the upper "+" sign in equations (2), the hyperbolic equations of the TDoA method correspond to the lower "-" sign in equations (2).

The method of linearization of hyperbolic equations is based on the expansion in a Taylor series in terms of the small parameter $r_i/r_C << 1$ and taking into account only the linear terms of the expansion. Such an approximation is performed with a high degree of accuracy in global positioning satellite systems, since the object is on the Earth and the base stations on the satellites are in space. However, for local positioning systems the parameter r_i/r_C is not small.

In this regard, we obtained exact solutions of hyperbolic equations in [8] on the plane and in [12] in space, and developed an analytical algorithm that eliminates spatial ambiguity zones in determining the coordinates of the target on the plane and in space with high accuracy in determining the coordinates of the object. In this paper, we propose a combined TSoA/TDoA method for determining the coordinates of an object on a plane.

After substitution of $X = x - x_C$, $Y = y - y_C$, the system of equations can be rewritten as:

$$\begin{cases} \sqrt{(X-X_L)^2+(Y-Y_L)^2} \pm \sqrt{X^2+Y^2} = L \\ \sqrt{(X-X_R)^2+(Y-Y_R)^2} \pm \sqrt{X^2+Y^2} = R \\ \sqrt{(X-X_U)^2+(Y-Y_U)^2} \pm \sqrt{X^2+Y^2} = U \end{cases}, \text{ where } \begin{matrix} X_L = x_L - x_C,\ Y_L = y_L - y_C,\ L = c\cdot\Delta\tau_{LC} \\ X_R = x_R - x_C,\ Y_R = y_R - y_C,\ R = c\cdot\Delta\tau_{RC}. \\ X_U = x_U - x_C,\ Y_U = y_U - y_C,\ U = c\cdot\Delta\tau_{UC} \end{matrix} \quad (3)$$

Let us denote $K = \sqrt{X^2+Y^2}$, where $K > 0$:

$$K^2 = X^2 + Y^2. \quad (4)$$

The system of Equations (5) can be rewritten as:

$$\begin{cases} \sqrt{(X-X_L)^2+(Y-Y_L)^2} = \mp K + L \\ \sqrt{(X-X_R)^2+(Y-Y_R)^2} = \mp K + R \\ \sqrt{(X-X_U)^2+(Y-Y_U)^2} = \mp K + U \end{cases}. \quad (5)$$

Squaring and reducing the general terms leads to the form:

$$\begin{cases} -2X_LX - 2Y_LY = \mp 2LK + L^2 - X_L^2 - Y_L^2 \\ -2X_RX - 2Y_RY = \mp 2RK + R^2 - X_R^2 - Y_R^2 \\ -2X_UX - 2Y_UY = \mp 2UK + U^2 - X_U^2 - Y_U^2 \end{cases}, \text{ or } \begin{cases} -2X_LX - 2Y_LY = E \mp 2LK \\ -2X_RX - 2Y_RY = F \mp 2RK \\ -2X_UX - 2Y_UY = G \mp 2UK \end{cases}, \quad (6)$$

where $E = L^2 - X_L^2 - Y_L^2;\ F = R^2 - X_R^2 - Y_R^2;\ G = U^2 - X_U^2 - Y_U^2.$

In matrix form, the three systems of equations are:

$$\begin{bmatrix} -2X_L & -2Y_L \\ -2X_R & -2Y_R \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} E \mp 2LK \\ F \mp 2RK \end{bmatrix},\quad \begin{bmatrix} -2X_R & -2Y_R \\ -2X_U & -2Y_U \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F \mp 2RK \\ G \mp 2UK \end{bmatrix},$$
$$\begin{bmatrix} -2X_U & -2Y_U \\ -2X_L & -2Y_L \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} G \mp 2UK \\ E \mp 2LK \end{bmatrix}. \quad (7)$$

Solution of the first system of equations ($i = 1$) is:

$$X_1 = \frac{1}{\Delta_1} \cdot \begin{vmatrix} E \mp 2LK & -2Y_L \\ F \mp 2RK & -2Y_R \end{vmatrix} = M_{1X} \cdot K + N_{1X}, \quad (8)$$

where $\Delta_1 = 4\left(X_L Y_R - Y_L X_R\right),\ M_{1X} = \frac{4}{\Delta_1}\cdot\left(\mp RY_L \pm LY_R\right),\ N_{1X} = \frac{2}{\Delta_1}\cdot\left(FY_L - EY_R\right);$

$$Y_1 = \frac{1}{\Delta_1}\cdot\begin{vmatrix} -2X_L & E \mp 2LK \\ -2X_R & E \mp 2RK \end{vmatrix} = M_{1Y}\cdot K + N_{1Y}, \quad (9)$$

where $M_{1Y} = \frac{4}{\Delta_1}\cdot\left(\mp LX_R \pm RX_L\right),\ N_{1Y} = \frac{2}{\Delta_1}\cdot\left(EX_R - FX_L\right)\cdot M_1;$

Solution of the second system of equations ($i = 2$) is:

$$X_2 = \frac{1}{\Delta_2}\cdot\begin{vmatrix} F \mp 2RK & -2Y_R \\ G \mp 2UK & -2Y_U \end{vmatrix} = M_{2X}\cdot K + N_{2X}, \quad (10)$$

where $\Delta_2 = 4\left(X_R Y_U - Y_R X_U\right),\ M_{2X} = \frac{4}{\Delta_2}\cdot\left(\mp UY_R \pm RY_U\right),\ N_{2X} = \frac{2}{\Delta_2}\cdot\left(GY_R - FY_U\right);$

$$Y_2 = \frac{1}{\Delta_2}\cdot\begin{vmatrix} -2X_R & F \mp 2RK \\ -2X_U & G \mp 2UK \end{vmatrix} = M_{2Y}\cdot K + N_{2Y}, \quad (11)$$

where $M_{2Y} = \frac{4}{\Delta_2}\cdot\left(\mp RX_U \pm UX_R\right),\ N_{2Y} = \frac{2}{\Delta_2}\cdot\left(FX_U - GX_R\right).$

Solution of the third system of equations ($i = 3$) is:

$$X_3 = \frac{1}{\Delta_3}\cdot\begin{vmatrix} G \mp 2UK & -2Y_U \\ E \mp 2LK & -2Y_L \end{vmatrix} = M_{3X}\cdot K + N_{3X}, \quad (12)$$

where $\Delta_3 = 4\left(X_U Y_L - Y_U X_L\right),\ M_{3X} = \frac{4}{\Delta_3}\cdot\left(\mp LY_U \pm UY_L\right),\ N_{3X} = \frac{2}{\Delta_3}\cdot\left(EY_U - GY_L\right);$

$$Y_3 = \frac{1}{\Delta_3}\cdot\begin{vmatrix} -2X_U & G \mp 2UK \\ -2X_L & E \mp 2LK \end{vmatrix} = M_{3Y}\cdot K + N_{3Y}, \quad (13)$$

where $M_{3Y} = \frac{4}{\Delta_3}\cdot\left(\mp UX_L \pm LX_U\right),\ N_{2Y} = \frac{2}{\Delta_3}\cdot\left(GX_L - EX_U\right).$

In expressions (2–13), at the summation sign “±”, the superscript corresponds to the solution of the elliptic TSoA equations, the subscript corresponds to the solution of the hyperbolic TDoA equations. Thus, equations (2–13) are solutions to both elliptic and hyperbolic equations.

Substitution of Equation (8–13) into Equation (4) defines a quadratic equation with respect to the variable K: $a_i K^2 + b_i K + c_i = 0$, where i corresponds to the choice of a pair of ellipses or hyperboles, $i = 1, 2, 3$:

$$a_i = M_{iX}^2 + M_{iY}^2 - 1,\quad b_i = 2\left(M_{iX}N_{iX} + M_{iY}N_{iY}\right),\quad c_i = N_{iX}^2 + N_{iY}^2. \quad (14)$$

The roots of the quadratic equation are the following $\left(b_i^2 - 4a_ic_i \geq 0\right)$:

$$K_{i1} = \frac{-b_i + \sqrt{b_i^2 - 4a_ic_i}}{2a_i}, \quad K_{i2} = \frac{-b_i + \sqrt{b_i^2 - 4a_ic_i}}{2a_i}. \tag{15}$$

In the case that one of the two roots K_{i1} and K_{i2} is negative for the same value of i, it can be immediately excluded from the solution, and then another root of the equation remains in the algorithm. However, a situation is possible when both roots K_{i1} and K_{i2} are positive. In this case, it becomes impossible to determine the coordinates of the object. It is for such a fairly common case that it is necessary to use the fourth BS in the positioning system on the plane.

Substitution of roots K_{i1} and K_{i2} in Equation (8–13) defines six possible sets of [*x*; *y*] coordinates of an object, defining six points in plane:

$$\left[x_{i1} = X_{i1} + x_C,\ y_{i1} = Y_{i1} + y_C\right],\ \left[x_{i2} = X_{i2} + x_C,\ y_{i2} = Y_{i2} + y_C\right], \tag{16}$$

where $X_{i1} = M_{iX} \cdot K_{i1} + N_{iX}$, $Y_{i1} = M_{iY} \cdot K_{i1} + N_{iY}$, $X_{i2} = M_{iX} \cdot K_{i2} + N_{iX}$, $Y_{i2} = M_{iY} \cdot K_{i2} + N_{iY}$.

In the absence of TDoA measurement errors from six calculated sets of [*x*; *y*] – coordinates, the coordinates of three points will be the same. It is this decision that is the true decision. To identify it, an algorithm was proposed in [8] and [12] based on finding the minimum distances between the intersection points of hyperboloids in space [12] and hyperbolas in the plane [8].

Simulation of TSoA and TDoA algorithms for estimating object coordinates with fluctuations in the measurement of arrival time

Arrival time measurement errors occur for three main reasons: 1) radio signal attenuation during propagation, 2) radio signal fading due to multipath propagation, 3) time desynchronization at base stations. In this work, the effect of base station time desynchronization on the accuracy of estimating object coordinates using a linearization algorithm and an exact analytical solution of elliptic and hyperbolic equations is studied. Using a computer experiment in the LabVIEW environment, the dependence of the root-mean square (RMS) deviation of the calculated object coordinates on the standard deviation of the difference in the times of arrival of radio signals at the *BS* was studied. To do this, Gaussian white noise with the same standard deviation for all *BS*s was added to the value of the arrival time at the input of each receiver of the positioning system. To determine the RMS deviation of the object position estimate, $n = 100$ computational experiments were carried out for each value of the standard deviation of the Gaussian white noise. The standard deviation of the estimate of the object's coordinates was determined in accordance with the expression:

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\left(x_i - x\right)^2 + \left(y_i - y\right)^2\right]}, \tag{17}$$

where index *i* – experiment number, [x_i; y_i] – estimation of object coordinates in the *i*-th experiment, [*x*; *y*] – true object coordinates.

As noted in [12], the accuracy of determining the coordinates strongly depends on the choice of the *BS* position. If only one reference base station is used in the scenario of the positioning system, gross errors occur in determining the coordinates of the object at points, which the determinant of the system of equations becomes zero. An example of such a situation is shown in Fig. 2. The calculation of the RMS deviation was carried out for the number of computational experiments 100, time of arrival deviation (TSoA and TDoA) 0.4 ns, base station coordinates *C*: (-40; 40), *L*: (-40; -40), *R*: (40; -40), *U*: (40; 40) meters.

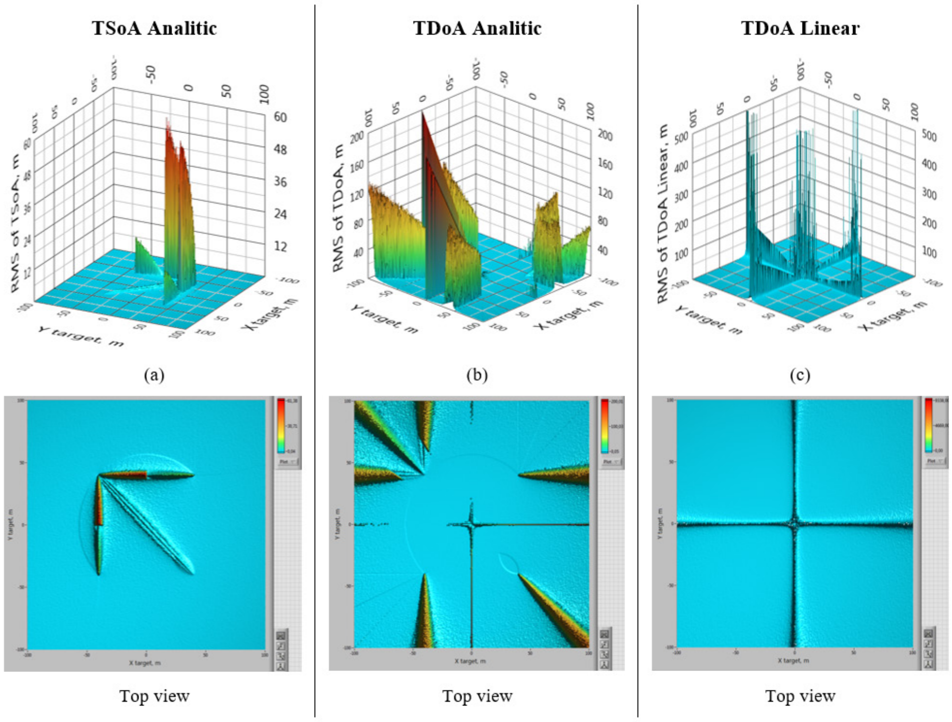

Fig. 2. Location of gross errors in determining the coordinates of an object on a plane for methods (a) TSoA Analitic, (b) TDoA Analitic, (c) TDoA Linear

Gross errors are common to all three methods under study. The errors in the TDoA Linear method are especially large. In addition, this method has significantly worse parameters of susceptibility to signal arrival time fluctuations, which was established in our previous studies [8, 12]. As can be seen from Fig. 2, the gross errors of all three methods have different positions on the plane. In the TSoA and TDoA methods, the positions of gross errors intersect only at the center of the BS location. Their position already suggests the idea of merging these two methods in order to eliminate gross errors associated with zeroing the determinants of systems of equations.

The RMS minima for determining object coordinates are achieved at line intersection angles (ellipses or hyperbolas) close to 90°. Fig. 3 shows the positions of the lines (ellipses and hyperbolas) at the minimum RMS values of the TSoA and TDoA methods with a Gaussian noise arrival time deviation of 0.4 ns. It is clearly seen that the true value of the position of the object is determined by the intersection of three lines. There are a significant number of points of intersection of the two lines. The algorithm proposed by us in [12] allows us to choose exactly the point of intersection of three lines from all the points of intersection of the lines.

Gross errors in determining the location of an object occur when the determinants of systems of equations are set to zero. Gross errors of the TSoA method are illustrated by the example shown in Fig. 4 a. The eccentricity of the ellipse L is equal to one, and in the presence of noise, the common triple point of intersection of the ellipses is constantly thrown between two closely spaced arcs of the ellipse L.

In the TDoA method, the number of failed configurations is doubled compared to the TSoA method. The first unsuccessful configuration corresponds to the eccentricity value equal to infinity ($\varepsilon = \infty$), the second unsuccessful configuration corresponds to the eccentricity value equal to one ($\varepsilon = 1$). Examples of both such configurations are shown in Fig. 4, b, c. At $\varepsilon = \infty$, both branches of the hyperbola merge into one straight line (Fig. 4, b) and the configuration is symmetrical about the horizontal axis. The TDoA method cannot determine the final intersection point: either this point is located at coordinates [0; -100] m, or at [0; +100].

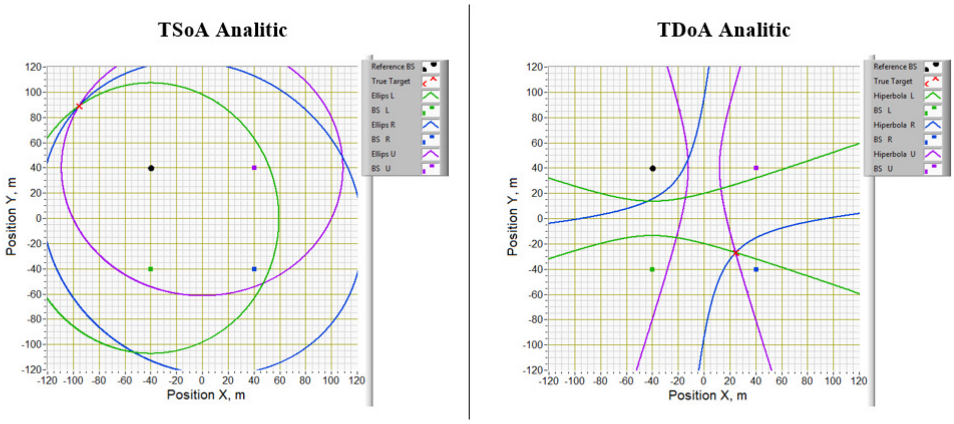

Fig. 3. Positions of ellipses and hyperbolas at minimum RMS values
(a) View of ellipses at minimum RMS TSoA 0.061 m, whereas TDoA method has RMS 45.8 m,
(b) View of hyperbolas at minimum RMS TDoA 0.058 m, whereas TSoA method has RMS 0.68 m

This type of error is the most significant, and can actually greatly harm the positioning system. It should be noted that the TSoA method copes well with this problematic configuration. For ε = 1, the branches of the hyperbola also merge into straight lines emerging from points with BS coordinates and diverging in different directions from the BS. The gross error is explained by rearranging the solution from one branch of the hyperbola to another. In this case, the errors are not as drastic as compared to the previous configuration, but they can also lead to unexpected consequences in a real system. The TSoA method works in this configuration as well. Thus, if the decision is correctly switched from the TDoA algorithm to the TSoA algorithm, depending on the configuration of the BS location and the object, blunders in many areas of the positioning system coverage can be eliminated. This is exactly what we offer in the combined TSoA/TDoA method.

The method proposed in [12] for hyperbolic equations can significantly reduce gross errors associated with the vanishing of determinants. The method consists in successive reassignment of each base station to the reference one. Thus, configurations are formed according to the number of BS in the system, in the case under consideration there will be four of them. All distances between the points of intersection of lines are recorded in a single 2D distance matrix, then three minimum distances between points in the distance matrix in all four configurations are determined and the point of intersection of three hyperbolas is determined from them. The method of reassigning each BS to the reference one made it possible to significantly reduce gross errors over the entire area of the positioning system operation. The TSoA method showed particularly good results. The root-mean-square deviation of the RMS maxima decreased by 60 times, leaving only one maximum in the center of the positioning system zone. The reassignment of the BS to the reference ones for the TDoA method made it possible to reduce the standard deviation of the RMS maxima by 2.5 times, to reduce the area of the positioning system service area, where gross errors occur. However, there are still zones with gross errors. That is, it turns out that the TDoA method is more accurate inside the BS perimeter, and the TSoA method is more accurate than the TDoA method outside the BS perimeter. So, the advantages of the TSoA method over the TDoA method include fewer gross errors and better accuracy outside the BS perimeter, and the advantage of TDoA over TSoA should include more accurate work inside the BS perimeter. The combined TSoA/TDoA method has the potential to combine the benefits of these two methods into one.

Simulation of the combined TSoA/TDoA method for estimating object coordinates with fluctuations in the measurement of arrival times

The combined TSoA/TDoA method assumes the choice of the best method for the accuracy of determining the coordinates and the absence of gross errors from TSoA or TDoA methods, depending on

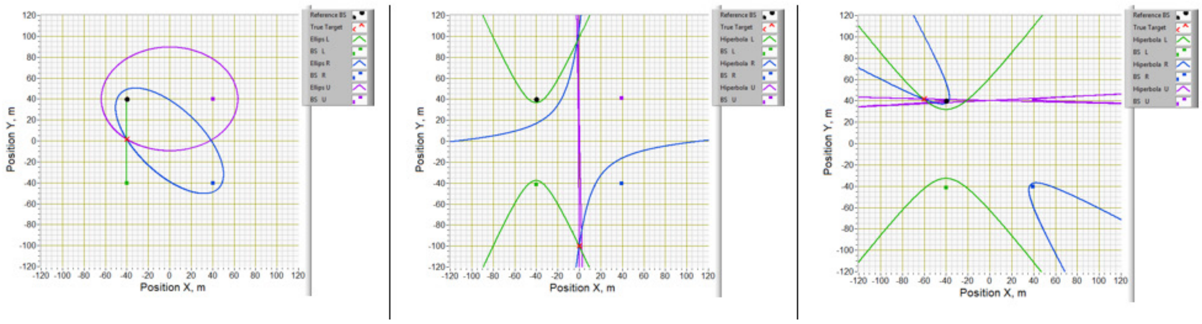

Fig. 4. Positions of ellipses and hyperbolas at maximum RMS values
(a) View of ellipses at maximum RMS TSoA 61.4 m, whereas TDoA method has RMS 0.09 m,
(b) View of hyperbolas at maximum RMS TDoA 200 m. The hyperbola *U* has an eccentricity $\varepsilon = \infty$,
whereas TSoA method has RMS 0.064 m, (c) View of hyperbolas at maximum RMS TDoA 46 m.
The hyperbola *U* has an eccentricity $\varepsilon = 1$, whereas TSoA method has RMS 0.061 m

the location of the BS and the object. The main idea of the proposed method is to combine the distances between the intersection points for both methods into a single 2D matrix and search for the minima of the distances between the intersection points of lines (ellipses and hyperbolas) from this single matrix. Thus, the algorithm chooses the TSoA or TDoA method. To identify the correct (true) point of intersection of the three lines, Algorithm 1 is proposed, which is an implementation of the combined TSoA / TDoA method.

Algorithm 1. Identifying of the true solution

1. Changing the reference BS according to 4 configurations of the spatial arrangement of the BS. Each BS once becomes a reference.

2. Formation of a 2D matrix of coordinates of the points of intersection of lines: each of the 4 lines (each line corresponds to the configuration of the BS) contains 12 coordinates of the points of intersection (6 points of intersection of ellipses and 6 points of intersection of hyperbolas).

3. Calculation of distances between the points of intersection of lines (6 points of ellipses and 6 points of hyperbolas). Between 6 points of intersection of lines, 15 distances between points are calculated for ellipses and 15 distances between points for hyperbolas.

4. Formation of a 2D distance matrix: 4 lines (each line corresponds to the BS configuration) contain 30 distances between the intersection points of lines: 15 distances between the intersection points of ellipses and 15 distances between the intersection points of hyperbolas. The distance table is common for the TSoA and TDoA methods, thus combining the two methods TSoA and TDoA.

5. Selection of 9 consecutive minima from a 2D distance table. Formation of 1D matrices for the values of 9 minimum distances, 9 configuration indices and 9 indices in the distance table.

6. Separation of 9 minimum distances, 9 configuration indices and 9 indices in the distance table on the basis of ellipses or hyperbolas. If the index in the distance table is less than 15, then this is the intersection point of the ellipses, if the index in the distance table is greater than or equal to 15, then this is the intersection point of the hyperbolas.

7. Selecting the BS configuration. If there are points of intersection of ellipses, then the configuration with ellipses is selected. If there are several intersection points of the ellipses, then the BS configuration with the minimum value of the distance D.

8. Selection of an index in the table of distances in a certain (item 7) configuration of the BS, corresponding to the minimum distance between the points of intersection of the lines.

9. According to the identified configuration index (item 7) and the distance index (item 8) from the 2D matrix of coordinates of the points of intersection of the lines (item 2), we select the coordinates of the object.

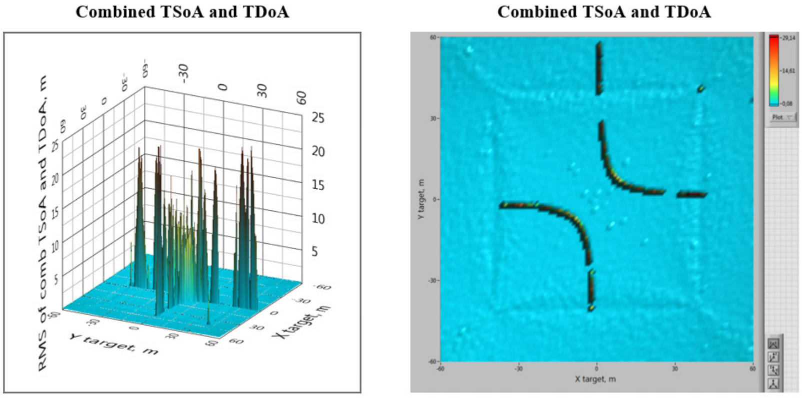

Fig. 5. The location of gross errors in determining the coordinates of the object on the plane of the combined method TSoA/TDoA (a) RMS of TSoA/TDoA, (b) Top view

The selected coordinates are closest to the true coordinates of the target. The remaining coordinates are either false, or determine the coordinates of the object with a larger error.

Fig. 5 shows the location of gross errors in determining the coordinates of an object on the plane of the combined TSoA/TDoA method, obtained using simulation. As before, the calculation of the root mean square (RMS) deviation was carried out for the number of computational experiments 100, the arrival time deviation was 0.4 ns.

The simulation results show that the combined TSoA/TDoA method makes it possible to reduce the RMS maxima by a factor of 4 compared to the TDoA method, and further reduce the area of the positioning system service area where gross errors occur. In areas where there are no gross errors, the RMS position determination coincides with the TDoA method, which, as indicated above, had advantages over the TSoA method. However, it should be noted that it was not possible to completely eliminate gross errors, although they significantly decreased.

Fig. 6 shows the dependences of the standard deviation of the estimate of the object's coordinates on the standard deviation of the Gaussian white noise of the times of arrival of radio signals in the BS for the matrix linearization algorithm and the proposed analytical combined TSoA/TDoA algorithm. Dependencies are calculated for object coordinates [x; y] = [39; -7] and BS_C coordinates $[x_C; н_C]$ = [- 40 ; 40], BS_L $[x_L; x_L]$ = [- 40 ; - 40], BS_R $[x_R; y_R]$ = [40 ; - 40], BS_U $[x_U; y_U]$ = [40 ; 40] meters. This location of the object and the BS is a rather complicated configuration for determining the coordinates, however, the combined method allows you to determine the coordinates of the object with high accuracy.

It follows from the simulation results that the proposed combined TSoA/TDoA method provides an 8 times higher accuracy in determining the object's coordinates compared to the hyperbolic equation linearization method. The combined TSoA/TDoA method works with high accuracy (~1 m) with BS desynchronization up to 8 ns, while the linearization method starts to give gross errors (failures) already at 4 ns BS desynchronization. The proposed algorithm significantly outperforms the widely used positioning algorithm based on the linearization of hyperbolic equations.

Discussion

The combined TSoA/TDoA method based on the analytical solution of elliptic and hyperbolic equations, unlike many existing methods, allows you to obtain an accurate solution for determining the coordinates of an object based on the measurement of arrival times. The advantages of the combined method are the high accuracy of determining the coordinates of the object, a significant reduction in the zones in which gross errors occur due to the vanishing of the determinants of the systems of equations. The meth-

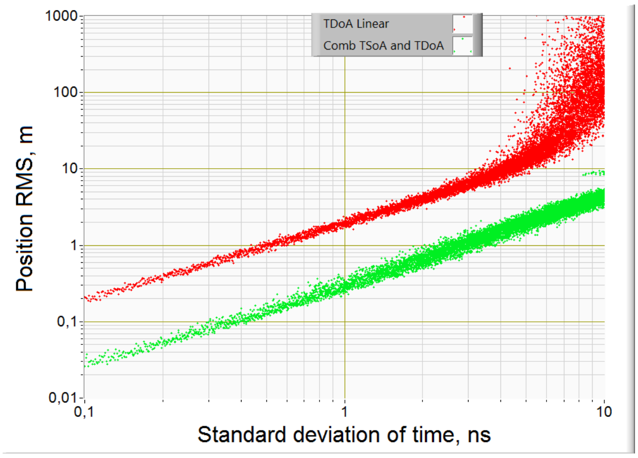

Fig. 6. Dependence of the RMS estimate of the determination of the object's coordinates on the plane versus the deviation of the Gaussian white noise of arrival times for the combined TSoA/TDoA and TDoA Linear methods

od allows you to automatically choose the best solution from elliptic and hyperbolic equations. A certain problem in the implementation and practical use of the combined method may be the limited capabilities of microprocessor technology for the implementation of calculations of expressions with radicals, which is necessary in the analytical exact method proposed by us. However, it should be noted that the computing element base is rapidly developing, the introduction of low-cost processors is expected, which allow calculating mathematical functions with radicals in real time. In many applications, such as "smart city" traffic control systems, all calculations can be performed on a computationally powerful server. In this example, restrictions on mathematical data processing are removed.

To accurately determine the moment of arrival of a radio signal from an object to base stations, various forms of radio signals are used. The wider the spectrum of the radio signal, the more precisely it is localized in time. Therefore, ultra-wideband (UWB) signals with a wide spectrum have advantages over signals with a shorter spectrum and are used in positioning systems. In our article, the question of the shape of the used radio signal is not the subject of research.

Time-of-arrival measurement errors occur for three main reasons: radio signal propagation attenuation, radio multipath, and time desynchronization at base stations. In this work, to assess the accuracy of determining the coordinates of an object, Gaussian white noise is used, which is added to the difference and the sum of the signal arrival times. Without diminishing the importance of studying the influence of multipath propagation and attenuation of a radio signal during propagation, we nevertheless believe that the approach we used to study noise immunity allows us to compare the TSoA and TDoA methods.

As a promising area of research, we consider the joint TSoA/TDoA method based on solving a system of equations by the intersection of a hyperbola with an ellipse. The joint TSoA/TDoA method will reduce the number of BSs in a multi-position system by one compared to the combined TSoA/TDoA method studied in this article.

In future work we intend to conduct a research on the effect of multipath propagation of a radio signal, take into account the attenuation of a radio signal during propagation, develop a mathematical model and investigate the joint TSoA/TDoA method, as well as extend the proposed algorithms to 3D space and consider the applicability of the proposed methods for determining the coordinates of a set of objects.

Conclusions

A feature of this work is the construction of a positioning system based on the solution of exact analytical equations, which eliminates the need to use iterative procedures when calculating the coordinates of an object. A comparative analysis of the advantages and disadvantages of the two main methods for determining the coordinates of an object, TSoA and TDoA, was carried out, and based on the analysis, a combined TSoA/TDoA method was proposed. As a result of the simulation, it was revealed that the combined TSoA/TDoA method has an accuracy of determining the coordinates of an object an order of magnitude higher than the frequently used method of linearizing hyperbolic equations. The proposed method will provide local positioning systems with the possibility of increasing the maximum detection and control range of objects with a low effective area scattering.

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Submitted: 11.05.2023; Approved: 25.06.2023; Accepted: 06.07.2023.
Поступила: 11.05.2023; Одобрена: 25.06.2023; Принята: 06.07.2023.