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COMPLETE SYNCHRONIZATION OF CHAOTIC SYSTEMS AT DIFFERENT PARAMETER VALUES

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Abstract. The article solves the problem of controlling the mode of complete synchronization of two chaotic systems at different values of their parameters. The control is based on the principle of linear feedback on the phase vector of chaotic generators. The introduction of feedback makes it possible to ensure the equality of the components of the phase vectors of the receiver and the transmitter due to the equality of their Lyapunov characteristic indicators. To change the Lyapunov spectrum, it is proposed to synthesize control by the modal control method based on the solution of the Sylvester linear matrix equation on the basis of the theorem on the topological equivalence of the nonlinear system and its linearized model. An example of using this technique to synchronize the chaotic oscillations of two Lorentz systems when transmitting information using chaotic masking is considered. Computational experiments confirm the operability of the proposed method of ensuring the synchronization of two chaotic systems.

Keywords: synchronization, chaotic generators, hidden data transmission system, chaotic masking, control with feedback

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ПОЛНАЯ СИНХРОНИЗАЦИЯ ХАОТИЧЕСКИХ СИСТЕМ ПРИ РАЗЛИЧНЫХ ЗНАЧЕНИЯХ ПАРАМЕТРОВ

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Аннотация. Изучена задача управления режимом полной синхронизации двух систем с хаотической динамикой и странным атрактором при различных значениях их параметров. Управление строится по принципу линейной обратной связи по фазовому вектору хаотических генераторов. Введение обратной связи позволяет обеспечить равенство компонент фазовых векторов приёмника и передатчика при равенстве их характеристических показателей Ляпунова. Для изменения спектра Ляпунова предложено на основе теоремы о топологической эквивалентности нелинейной системы и её линеаризованной модели в гиперболическом случае синтезировать управление методом модального управления на основе решения линейного матричного уравнения Сильвестра. Рассмотрен пример применения данной методики для синхронизации хаотических колебаний двух систем Лоренца при передаче информации с использованием хаотической маскировки. Вычислительные эксперименты подтверждают работоспособность предлагаемого метода на основе равенства одноименных компонент фазовых векторов передатчика и приёмника и соответствующих значений критериев полной синхронизации двух хаотических систем.

Ключевые слова: синхронизация, хаотические генераторы, система скрытой передачи данных, хаотическое маскирование, управление с обратной связью

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Introduction

At present, there is an increased interest of Russian and foreign authors in theoretical research and practical application of nonlinear systems with chaotic modes, which is caused by the widespread introduction of information technologies in various spheres of civil and military application [1–4]. One of the important problems in the creation of information systems is the problem of ensuring reliable and confidential communication [5].

One of the promising directions in the development of telecommunication systems is the use of deterministic chaos [6, 7], which allows providing a high level of protection and a wide band of the carrier with its large information capacity [8, 9]. The main issues in the construction of telecommunication systems based on chaotic generators are issues related to the synchronization of chaotic generators. To solve these problems, various types of synchronization are used: generalized synchronization [10–12], frequency synchronization [13], and phase synchronization [13–15].

Among the various systems of covert information transmission [16–19], systems with chaotic masking based on full synchronization have become widespread [20, 21]. To implement full synchronization, various approaches are used on the basis of rough chaotic systems [22, 23], hyperchaotic systems [24, 25], and the introduction of mutual influence between two chaotic generators [26, 27].

The paper proposes a method to ensure full synchronization of chaotic generators with different values of parameters caused by technological reasons. The considered method is based on the management of the spectrum of characteristic indicators of Lyapunov chaotic generators [28, 29].

The problem of complete synchronization of two generators with chaotic dynamics

Mathematical model of chaotic generators. Let the dynamics of a system of hidden information transmission, which consists of a transmitter, a receiver, and a communication line, be described by two autonomous families of ordinary differential equations

$$\dot{X}(t) = dX(t)/dt = F\big(X(t),\ \alpha,\ u^1(t)\big) + G\big(Z(t),\ X(t)\big),\ X(0) = X_0, \tag{1}$$

$$\dot{Z}(t) = dZ(t)/dt = F\big(Z(t),\ \beta,\ u^2(t)\big) + G\big(X(t),\ Z(t)\big),\ Z(0) = Z_0, \tag{2}$$

where $X(t) \in \mathrm{R}^n$ is the vector of the transmitter state, $Z(t) \in \mathrm{R}^n$ is the vector of the receiver state, $\alpha \in \mathrm{R}^s$ is the vector of the transmitter parameters, $\beta \in \mathrm{R}^s$ is the vector of the receiver parameters, $u^1(t),\ u^2(t) \in \mathrm{R}^m$ are the vectors of the transmitter and receiver control, $m \le n$, $F : \mathrm{R}^n \otimes \mathrm{R}^s \otimes \mathrm{R}^m \to$ $\to \mathrm{R}^n$ is a vector function that determines the dynamics of the transmitter and receiver behavior, $G : \mathrm{R}^n \otimes \mathrm{R}^n \to \mathrm{R}^n$ is a vector function that determines the nature of the communication between the transmitter and the receiver. The function G depends linearly on the state vector of the transmitter and receiver.

The vector function F has a given smoothness class by vector arguments $X(t),\ Z(t)$, and $u^i(t)$, that is $F \in \mathbb{C}^r_X$, $F \in \mathbb{C}^r_Z$, and $F \in \mathbb{C}^r_{u^i}$, or satisfies the Lipschitz condition

$$\left\|F(X') - F(X'')\right\| \le k\left\|X' - X''\right\|,\ \left\|F(Z') - F(Z'')\right\| \le k\left\|Z' - Z''\right\|,\ k > 0.$$

The vector function F:

1) is unstable in relation to the setting of initial conditions – there is a value δ, such that for an arbitrary point $X \in \mathrm{R}^n$ and $\varepsilon > 0$, there is a point $Y \in \mathrm{R}^n$ for which the condition $\mathrm{dist}\big[X(t_0),\ Y(t_0)\big] < \varepsilon$ follows $\mathrm{dist}\big[X(t),\ Y(t)\big] \ge \delta$ for some $t > t_0$, where $\mathrm{dist}[*,*]$ is the distance;

2) is topologically transitive – for any two open sets N, M, there is such l that $F^l(N) \cap M \ne \varnothing$;

3) has the element of regularity or otherwise density of periodic trajectories – in any vicinity of any point of phase space there is at least one and, therefore, infinitely many periodic trajectories.

These three conditions determine the presence of chaotic dynamics in systems (1) and (2) at certain values of their parameters.

Criteria for complete synchronization of chaotic systems. Complete synchronization of chaotic systems occurs when the evolution in time of the states of the interacting chaotic generators proceeds in the same way after the completion of the transient process. In this case, the equality of the similar components of the vectors of the transmitter and receiver state is ensured:

$$x_1(t) = z_1(t), \ldots, x_i(t) = z_i(t), \ldots, x_n(t) = z_n(t). \tag{3}$$

Various criteria are used to assess the full synchronization mode [30].

Synchronization error for each time point

$$x_1(t) - z_1(t), \ldots, x_i(t) - z_i(t), \ldots, x_n(t) - z_n(t). \tag{4}$$

The integral error

$$e = \int_0^\infty \left\| X(t) - Z(t) \right\| \mathrm{dt}. \tag{5}$$

The similarity function

$$G^2(\tau) = \frac{\left\langle \left(z_i(t+\tau) - x_i(t) \right)^2 \right\rangle}{\sqrt{\left(\left\langle x_i^2(t) \right\rangle \left\langle z_i^2(t) \right\rangle \right)}}, \tag{6}$$

where $x_i(t)$ and $z_i(t)$ are the same-name components of the phase vectors of the transmitter and receiver. In the case of full synchronization, the similarity function is zero.

Evaluation of the degree of synchronization

$$\upsilon_i = \frac{\sigma^2_{x_i+z_i}}{2\left(\sigma^2_{x_i} + \sigma^2_{z_i}\right)}, \cdots i = \overline{1,n}, \tag{7}$$

where $x_i(t)$ and $z_i(t)$ are the same-name state variables of the transmitter and receiver generators, $\sigma^2_{x_i}$ and $\sigma^2_{z_i}$ are the variance of the variables $x_i(t)$ and $z_i(t)$, $\sigma_{x_i+z_i}$ is the variance of their sum. The estimation of the degree of synchronization can take values $\upsilon_i \in [0.5;1]$, while $\upsilon_i = 1$ corresponds to the case of complete synchronization, and with the independence of the processes $x_i(t)$ and $z_i(t)$, we have .

The cross-correlation coefficient

$$\rho(x_i, z_i) = \frac{\dfrac{\sum_{j=1}^{K} \left(x_{ij} - \bar{X}\right)\left(z_{ij} - \bar{Z}\right)}{K}}{\sqrt{\dfrac{\sum_{j=1}^{K} \left(x_{ij} - \bar{X}\right)}{K}} \sqrt{\dfrac{\sum_{j=1}^{K} \left(z_{ij} - \bar{Z}\right)}{K}}}, \tag{8}$$

$$\bar{X} = \frac{\sum_{j=1}^{K} x_{ij}}{K}, \quad \bar{Z} = \frac{\sum_{j=1}^{K} z_{ij}}{K},$$

where x_{ij}, z_{ij} are the values of the similar components of the phase vectors of the transmitter and receiver at time points $j = \overline{j_o, j_k}$; $X = \{x_1, x_2, ..., x_j, ...\}$, $Z = \{z_1, z_2, ..., z_j, ...\}$ are a number of values of the components of the phase vectors of the transmitter and receiver.

The cross-correlation coefficient is equal to one when fully synchronized and tends to zero when there is no statistical relationship between the processes in the transmitter and receiver.

Full synchronization management task. At the same values of the parameters of chaotic generators, a full synchronization mode is observed. When changing the values of the parameters of one of the generators, desynchronization occurs and the identity of chaotic oscillations is lost, which disrupts the process of data transmission in the communication system built on the basis of these generators.

In this situation, the task arises of synthesizing control that restores the identity of chaotic oscillations $X(t) \equiv Z(t)$, $t > t_0$ at different values of transmitter and receiver parameters $\alpha \neq \beta$.

It is necessary to synthesize control in the form of linear stationary feedback on the phase vector of the transmitter and receiver when changing the parameters

$$u^1(t) = L^1 X(t);\ u^2(t) = L^2 Z(t). \tag{9}$$

Feedback coefficients L^i, $i = 1,2$ (9) are calculated based on the condition of equality of the spectrum of Lyapunov characteristic exponents of the receiver chaotic generator

$$\Omega\left(F,\ Z,\ \beta,\ u^2\right) = \left\{\chi_1, ..., \chi_i, ..., \chi_n : \chi_1 > \chi_i\ \forall i = \overline{2,n}\right\} \tag{10}$$

to the Lyapunov spectrum of the transmitter

$$\Omega\left(F,\ X,\ \alpha,\ u^1\right). \tag{11}$$

The equality of Lyapunov spectra makes it possible to ensure the identity of chaotic oscillations of generators and to ensure complete synchronization, which can be checked according to criteria (3)–(8).

Synthesis of the Lyapunov characteristic exponents spectrum control

Topological equivalence of nonlinear systems. The feedback control synthesis technique is based on the topological equivalence of the behavior of nonlinear systems to the behavior of a linearized system. The systems of nonlinear autonomous differential equations (1), (2) correspond to a vector field F. According to the Hartman–Grobman theorem [31, 32], a continuously differentiable vector field with a hyperbolic special point in some vicinity of this point is topologically equivalent to its linear part.

From this theorem, it follows that the qualitative behavior of the solutions to an autonomous system of nonlinear differential equations (1), (2) at a hyperbolic special point is completely determined by the behavior of the solutions to a system of linear differential equations with a constant operator (Jacobi matrix) of the linear part of the field at this point. At the hyperbolic special point of the nonlinear system, no Jacobi matrix eigenvalue lies on the imaginary axis.

The nature of the behavior of the solutions to linear differential equations corresponding to the nonlinear system is determined by the intrinsic values of the Jacobi matrix, the real parts of which are associated with its Lyapunov characteristic exponents. A characteristic exponent of a function $y(t)$ is a value that has a finite value or value $\pm\infty$ and is defined as

$$\lambda(y) \equiv \overline{\lim_{t\to\infty}}\left(t^{-1} \ln\left\|y(t)\right\|\right).$$

The characteristic exponent determines the change in the function according to the scale of indicative functions.

The nature of the behavior of solutions to nonlinear differential equations is determined by the Lyapunov spectrum. Negative characteristic exponents correspond to regular dynamics, in the presence of positive, negative and zero characteristic exponents in the Lyapunov spectrum, there is a chaotic dynamic.

Using one of the modal control methods, it is possible to provide the required spectrum of a linearized system, and by virtue of the theorem of topological equivalence – also the corresponding spectrum of a nonlinear system.

Linearization of a nonlinear system. Using the Taylor formula in the assumption of the smoothness of the vector function F by vector arguments $X(t)$, $Z(t)$, $u^i(t)\left(F \in \mathbb{C}^r_X,\ F \in \mathbb{C}^r_{u^i},\ \text{and}\ F \in \mathbb{C}^r_{u^i}\right)$ in the vicinity of a special point and limited to the linear term of the Taylor series, we convert equation (1) and (2) to the quasi-linear form:

$$\dot{X}(t) = J(X^S)X(t) + f(X^S), \quad X(0) = X_0, \tag{12}$$

$$\dot{Z}(t) = J(Z^S)Z(t) + f(Z^S), \quad Z(0) = Z_0. \tag{13}$$

In equations (12), (13), the Jacobian matrices $J(X^S)$ and $J(Z^S)$ are calculated at singular points X^S, Z^S of nonlinear systems (1) and (2) according to the following formulas:

$$J(X^S) = \begin{bmatrix} \partial f_1/\partial x_1 & \dots & \partial f_1/\partial x_n \\ \dots & \dots & \dots \\ \partial f_n/\partial x_1 & \dots & \partial f_n/\partial x_n \end{bmatrix}_{X(t)=X^S}, \tag{14}$$

$$J(Z^S) = \begin{bmatrix} \partial f_1/\partial z_1 & \dots & \partial f_1/\partial z_n \\ \dots & \dots & \dots \\ \partial f_n/\partial z_1 & \dots & \partial f_n/\partial z_n \end{bmatrix}_{Z(t)=Z^S}. \tag{15}$$

If all singular points are evaluated

$$\left\| f(\xi^S) \right\| \le q \left\| \xi \right\|,$$

and Jacobian matrices are calculated by formulas (14) and (15), then equations (12) and (13) will take the form of linearized systems (or equations in variations):

$$\dot{Y}(t) = J(X^s)Y(t), \tag{16}$$

$$\dot{W}(t) = J(Z^S)W(t). \tag{17}$$

Systems (16) and (17) can be used to synthesize control of nonlinear systems (1) and (2) in order to fully synchronize chaotic oscillations of receiver and transmitter generators.

Synthesis of control of complete synchronization of chaotic systems. Control of nonlinear systems (1) and (2) by introducing feedback consists in changing the spectrum of Lyapunov characteristic exponents to achieve the desired result – full synchronization of two chaotic systems at different values of parameters.

Using the modal control method based on the solution of the matrix algebraic Sylvester equation for a linearized system of a chaotic generator

$$\dot{W}(t) = J(Z^S)W(t) + Bu(t), \tag{18}$$

control in the form of static linear feedback

$$u(t) = LW(t) \tag{19}$$

is derived from the condition of equality of the spectrum of the closed system (18), (19)

$$\dot{W}(t) = J(Z^S)W(t) + BLW(t) = A_Z W(t), \tag{20}$$

$$A_Z = J(Z^S) + BL \tag{21}$$

to the required spectrum

$$\rho(A_Z) = \rho(\Phi).$$

Equation (18) is a linearized model of the nonlinear part of systems (1) and (2), according to which the decentralized feedback (19) is calculated from the state vector of the transmitter and receiver. Equations (18)–(20) are written in the coordinates of a linearized system and do not contain a mutual influence function G.

Let us represent the matrix (20) of a closed system by the expansion in the basis of its eigenvectors:

$$A_Z = S\Phi S^{-1}, \tag{22}$$

where Φ is the Jordan form of the matrix A_Z; S is the matrix of eigenvectors of the matrix A_Z. Then substituting (22) into (21) and multiplying by S, we get:

$$J(Z^S)S - S\Phi = -BLS. \tag{23}$$

Let us eliminate the nonlinear component in the right-hand side of equation (23) by introducing a matrix factor $V = -LS$, then the equation (23) will take the form:

$$J(Z^S)S - S\Phi = BV. \tag{24}$$

Equation (24) is the Sylvester matrix equation. It is linear with respect to S and solvable if the following conditions are met [33]:

1) full rank matrix V;
2) rank$D_u = n$, where $D_u = \left(B \middle| J(Z^S)B \middle| \ldots \middle| J(Z^S)^{n-1} B\right) \in R^{n \times mn}$;
3) rank$H_l = n$, where $H_l = \left(V^T \middle| \Phi^T V^T \middle| \ldots \middle| (\Phi^T)^{n-1} V^T\right) \in R^{n \times mn}$;
4) the spectra of matrices $J(Z^S)$ and Φ do not intersect;
5) the eigenvalues of the matrix Φ are pairwise different.

Having solved equation (24) with respect to the matrix S, one can calculate the matrix of feedback coefficients:

$$L = -VS^{-1}. \tag{25}$$

The matrix L found by formula (25) is used to control the Lyapunov spectrum of a chaotic generator on the transmitting and receiving sides:

$$\dot{X}(t) = dX(t)/dt = F(X(t), \alpha) + BL^1 X(t), \quad X(0) = X_0, \tag{26a}$$

$$\dot{Z}(t) = dZ(t)/dt = F(Z(t), \beta) + BL^2 Z(t), \quad Z(0) = Z_0, \tag{26b}$$

which ensures that receiver and transmitter performance is equal for complete synchronization.

Investigation of the information transmission system using chaotic masking

Examination of the system at the same values of transmitter and receiver parameters. The Lorentz system is considered as chaotic generators (1), (2) of the transmitter and receiver for the system with chaotic masking [34]:

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = x_1(r - x_3) - x_2, \\ \dot{x}_3 = x_1 x_2 - b x_3 \end{cases} \tag{27}$$

with parameters $\alpha = (\alpha_1 = \sigma = 12;\ \alpha_2 = b = 2;\ \alpha_3 = r = 45)$.

System (27) has three singular points:

$$X^{s=1} = (x_1 = 0;\ x_2 = 0;\ x_3 = 0)^{\mathrm{T}},$$
$$X^{s=2} = (x_1 = -9.3808;\ x_2 = -9.3808;\ x_3 = 44.0000)^{\mathrm{T}},$$
$$X^{s=3} = (x_1 = 9.3808;\ x_2 = 9.3808;\ x_3 = 44.000)^{\mathrm{T}}$$

and a Jacobian matrix,

$$J = \begin{bmatrix} -12 & 12 & 0 \\ 45 - x_3 & -1 & -x_1 \\ x_2 & x_1 & -2 \end{bmatrix},$$

the eigenvalues of which, calculated at the specified parameters at special points, are equal to

$$\rho\{\lambda_i J(X^{s=1}),\ i = \overline{1,3}\} = \begin{cases} \lambda_1 = -30.3799, \\ \lambda_2 = 7.3799, \\ \lambda_3 = -2.0000. \end{cases}$$

$$\rho\{\lambda_i J(X^{s=2}),\ i = \overline{1,3}\} = \begin{cases} \lambda_1 = -16.0791 + 0.0000j, \\ \lambda_2 = 0.5395 + 11.4481j, \\ \lambda_3 = 0.5395 - 11.4481j. \end{cases}$$

The Lyapunov spectrum of system (27)

$$\Omega(F, X, \alpha) = \{\chi_1 = 0.6283;\ \chi_2 = 0.4156;\ \chi_3 = -16.0441\}$$

contains a positive characteristic indicator, and the phase volume of the system is compressed, so a chaotic mode occurs in the Lorentz system at the specified parameters. The phase portrait of the system is shown in Fig. 1.

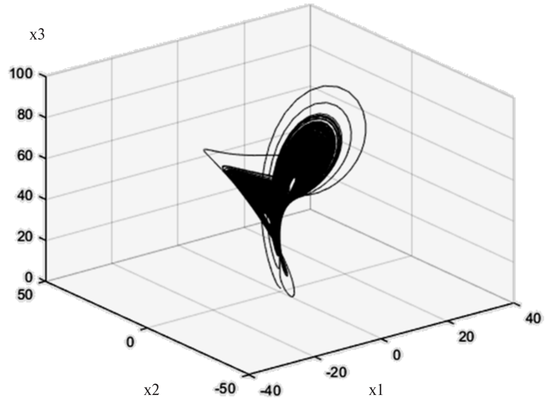

Fig. 1. Phase portrait of the Lorentz system

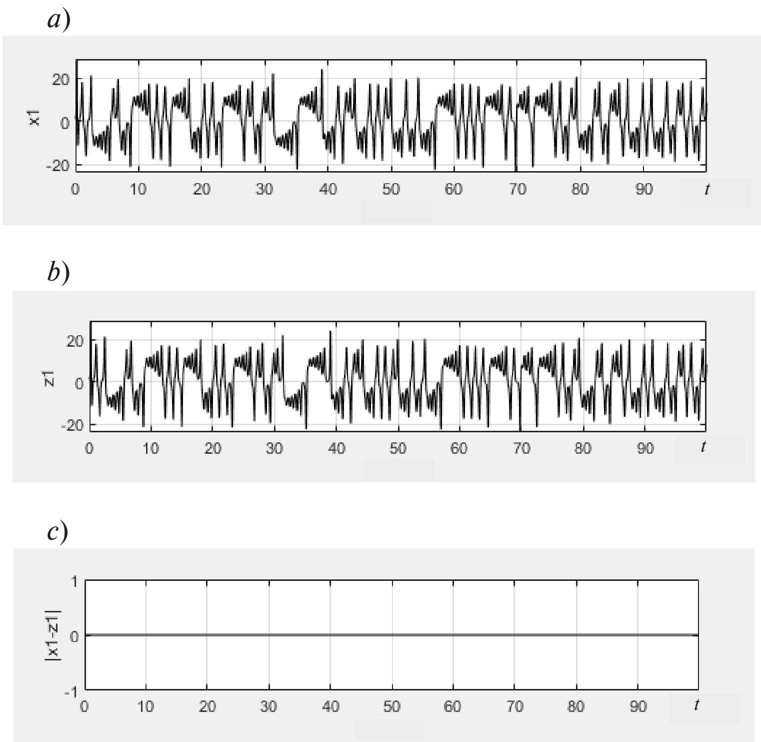

Fig. 2. Time diagrams with the same parameters of chaotic systems:
a – transmitter component x_1; *b* – receiver component z_1; *c* – component difference $(x_1 - z_1)$

Fig. 2 shows time diagrams of the first components of the phase vector of the transmitter and receiver and their difference at the same values of parameters.

Since the parameters of the transmitter and receiver are equal ($\alpha = \beta$), all the given characteristics are equal. Therefore, there is a complete synchronization of chaotic oscillations in the system. The values of the complete synchronization criteria are given in Table 1.

Examination of the system at different values of transmitter and receiver parameters. At different parameter values, the identity of the phase vectors is violated, which leads to the desynchronization of the receiver and transmitter. This effect is considered when the receiver parameter values are changed:

$$\beta = (\beta_1 = \sigma = 12.01;\ \beta_2 = b = 2.01;\ \beta_3 = r = 45) \neq$$
$$\neq \alpha = (\alpha_1 = \sigma = 12.00;\ \alpha_2 = b = 2.00;\ \alpha_3 = r = 45).$$

With the changed values of the parameters, the Lorentz system on the receiving side

$$\begin{aligned} \dot{z}_1 &= -z_2 - z_3 \\ \dot{z}_2 &= z_1 + az_2 \\ \dot{z}_3 &= b + z_3(z_1 - c), \end{aligned}$$

has the following characteristics:

– three singular points

$$\begin{aligned} X^{s=1} &= (x_1 = 0;\ x_2 = 0;\ x_3 = 0)^{\mathrm{T}}, \\ X^{s=2} &= (x_1 = -9.4043;\ x_2 = -9.4043;\ x_3 = 44.0000)^{\mathrm{T}}, \\ X^{s=3} &= (x_1 = 9.4043;\ x_2 = 9.4043;\ x_3 = 44.0000)^{\mathrm{T}}; \end{aligned}$$

– Jacobian matrix eigenvalues

$$\rho\left\{\lambda_i J\left(X^{s=1}\right),\ i = \overline{1,3}\right\} = \begin{cases} \lambda_1 = -30.3955, \\ \lambda_2 = 17.3855, \\ \lambda_3 = -2.0100; \end{cases}$$

$$\rho\left\{\lambda_i J\left(X^{s=2}\right),\ i = \overline{1,3}\right\} = \begin{cases} \lambda_1 = -16.0987 + 0.0000j, \\ \lambda_2 = +0.5394 + 11.4746j, \\ \lambda_3 = +0.5394 - 11.4746j; \end{cases}$$

– Lyapunov spectrum

$$\Omega(F, X, \alpha) = \{\chi_1 = 0.6284;\ \chi_2 = 0.4156;\ \chi_3 = -16.0441\}.$$

Fig. 3 shows time diagrams of the first components of the phase vector of the transmitter and receiver and their difference at different values of parameters.

There is a significant difference between the components of the same phase vectors in the system, and data transmission becomes impossible. The values of the complete synchronization criteria (actually de-synchronization) are given in Table 1.

Synchronization when there is mutual influence. To restore synchronization between the transmitter and the receiver, the mutual influence between the subsystems is introduced [35]. The system of two Lorentz oscillators with different parameters and with the addition of a coupling coefficient is described by equations of the form:

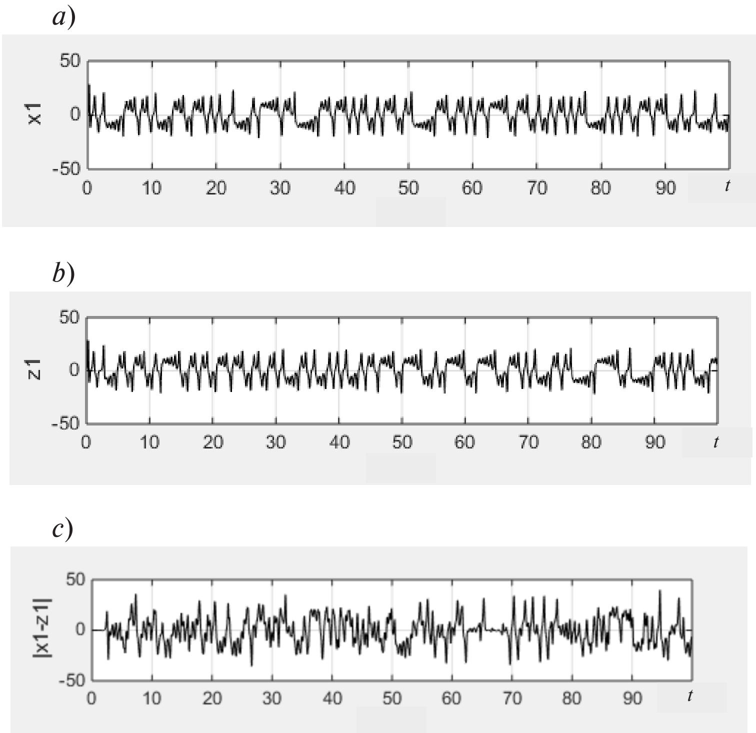

Fig. 3. Time diagrams with different parameters of chaotic systems:
a — transmitter component x_1; *b* — receiver component z_1; *c* — component difference $(x_1 - z_1)$

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1) + \gamma(z_1 - x_1), \\ \dot{x}_2 = x_1(r - x_3) - x_2, \\ \dot{x}_3 = x_1 x_2 - b x_3, \\ \dot{z}_1 = \sigma_2(z_2 - z_1) + \gamma(x_1 - z_1) \\ \dot{z}_2 = z_1(r_2 - z_3) - z_2, \\ \dot{z}_3 = z_1 z_2 - b_2 z_3. \end{cases} \tag{28}$$

The mutual influence function is determined by the difference of the first coordinates of the transmitter and receiver multiplied by a constant coefficient. With the parameter of interconnection $\gamma = 10$, the system (28) has the following Lyapunov characteristic exponents:

$$\Omega(F, X, Z, \alpha, \beta, \gamma) = \{\chi_1 = 0.8520;\ \chi_2 = 0.4764;\ \chi_3 = -0.2983;\ \chi_4 = -0.5547;\ \chi_5 = -15.8508;\ \chi_6 = -33.6646\}.$$

Fig. 4 shows time diagrams of the first components of the phase vector of the transmitter and receiver and their difference at different values of parameters and the presence of mutual connection between them.

The difference between the similar components of the phase vectors of the transmitter and the receiver is significantly reduced, which indicates the restoration of synchronization. The latter confirms the values of the synchronization criteria given in Table 1.

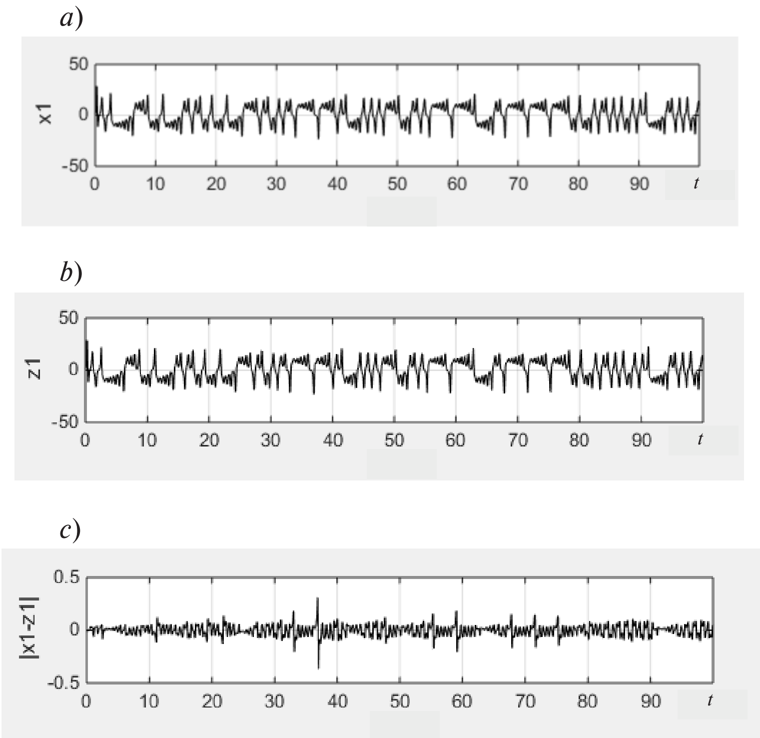

Fig. 4. Time diagrams with different parameters of chaotic systems and their interconnection: *a* – transmitter component x_1; *b* – receiver component z_1; *c* – component difference $(x_1 - z_1)$

The complete synchronization in the presence of mutual influence and management of the spectrum of characteristic exponents. To ensure the full synchronization mode of two chaotic generators at different values of parameters, feedback (19) is introduced into the Lorentz system of the transmitter and receiver and the system of autonomous differential equations (26) or (28) takes the form:

$$\begin{cases} \dot{x}_1 = \sigma\left(x_2 - x_1\right) - \left(l_{11}^1 x_1 + l_{12}^1 x_2 + l_{13}^1 x_3\right) + \gamma\left(z_1 - x_1\right), \\ \dot{x}_2 = x_1\left(r - x_3\right) - x_2 - \left(l_{21}^1 x_1 + l_{22}^1 x_2 + l_{23}^1 x_3\right), \\ \dot{x}_3 = x_1 x_2 - b x_3 - \left(l_{31}^1 x_1 + l_{32}^1 x_2 + l_{33}^1 x_3\right), \\ \dot{z}_1 = \sigma_2\left(z_2 - z_1\right) - \left(l_{11}^2 z_1 + l_{12}^2 z_2 + l_{13}^2 z_3\right) + \gamma\left(x_1 - z_1\right), \\ \dot{z}_2 = z_1\left(r_2 - z_3\right) - z_2 - \left(l_{21}^2 z_1 + l_{22}^2 z_2 + l_{23}^2 z_3\right), \\ \dot{z}_3 = z_1 z_2 - b_2 z_3 - \left(l_{31}^2 z_1 + l_{32}^2 z_2 + l_{33}^2 z_3\right). \end{cases} \tag{29}$$

Feedback coefficients L^i, $i = 1,2$ are derived according to formula (25) and are equal to:

$$L^1 = \begin{bmatrix} 0.1203 & -0.0489 & 0 \\ -0.1835 & -0.1203 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}; \quad L^2 = \begin{bmatrix} 0.1047 & -0.0426 & 0 \\ -0.1749 & -0.1147 & 0 \\ 0 & 0 & 0.49 \end{bmatrix}. \tag{30}$$

In formula (25), the matrix S is the solution to the Sylvester equation (24), $J^1(Z^S)$, $J^2(Z^S)$ is the solution to the Jacobian matrix of linearized models of the transmitter and receiver, $\Phi = \mathrm{diag}\{\chi_1 = 0.0709;\ \chi_2 = 0.0184;\ \chi_3 = -5.4453\}$ is the matrix of the required eigenvalues of the closed linearized system of the form (20).

The feedback system (29), (30) has the following Lyapunov characteristic exponents:

$$\Omega\left(F, X, Z, \sigma^1, \sigma^2, b^1, b^2 r\right) =$$
$$= \{\chi_1 = 0.8704;\ \chi_2 = 0.3316;\ \chi_3 = -16.7021;$$
$$\chi_4 = 0.8157;\ \chi_5 = 0.3706;\ \chi_6 = -16.6863\}.$$

Fig. 5 shows time diagrams of the first components of the phase vector of the transmitter and receiver and their difference with different parameters, the presence of mutual influence and control of the Lyapunov spectrum.

Thus, the control of the spectrum of Lyapunov characteristic exponents makes it possible to significantly reduce the difference between the similar components of the phase vectors of the transmitter and receiver, and the system is in the complete synchronization mode. The values of the full synchronization criteria, for this case, are given in Table 1.

Table 1

Criteria for complete synchronization of chaotic systems

Characteristics of the data transmission system	Criteria for complete synchronization								
	Dispersion coefficient			Correlation coefficient			Similarity function		
	$\nu_{x1,z1}$	$\nu_{x2,z2}$	$\nu_{x3,z3}$	$R_{x1,z1}$	$R_{x2,z2}$	$R_{x3,z3}$	$G_{x1,z1}$	$G_{x2,z2}$	$G_{x3,z3}$
Same transmitter and receiver parameters	1	1	1	1	1	1	0	0	0
Different transmitter and receiver parameters	0.4977	0.5020	0.4664	0.0047	0.0041	0.0673	0.0112	0.0106	0.0025
Existence of mutual influence	1	0.9999	0.9999	1	0.9999	0.9998	$1.5\ 10^{-5}$	$3.5\cdot10^{-5}$	0.0012
Existence of mutual influence and control of Lyapunov spectrum	1	1	1	1	1	1	$3.3\cdot10^{-6}$	$1.3\cdot10^{-5}$	$6.1\cdot10^{-4}$

In data transmission systems with chaotic masking, the signal at the transmitter output $S(t)$ is formed as a sum of transmitted information $m(t)$ and the value of the first component of the phase vector of the chaotic oscillator $x_1(t)$, that is $S(t) = m(t) + x_1(t)$. The signal at the output of the system $m(t)$ represents the difference between the signal transmitted over the communication line $S(t)$ and the value of the first component of the phase vector of the chaotic generator of the receiver $z_1(t)$, i.e. $m(t) = S(t) - z_1(t)$. Fig. 6 shows time diagrams of the information transmission system with chaotic masking.

Thus, with equal values of transmitter and receiver parameters, there is a complete synchronization of chaotic generators in the system. With different parameters, the process becomes out of sync. The introduction of mutual influence and feedback into the system allows restoring complete synchronization of chaotic signals with different parameters of the transmitter and receiver.

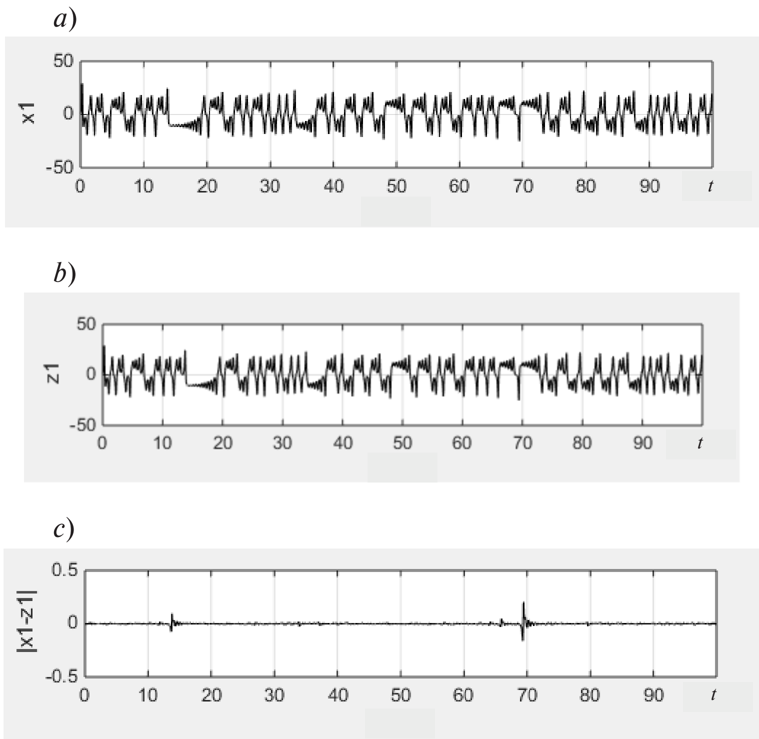

Fig. 5. Time diagrams with different parameters, mutual influence and control of Lyapunov spectrum: *a* – transmitter component x_1; *b* – receiver component z_1; *c* – component difference $(x_1 - z_1)$

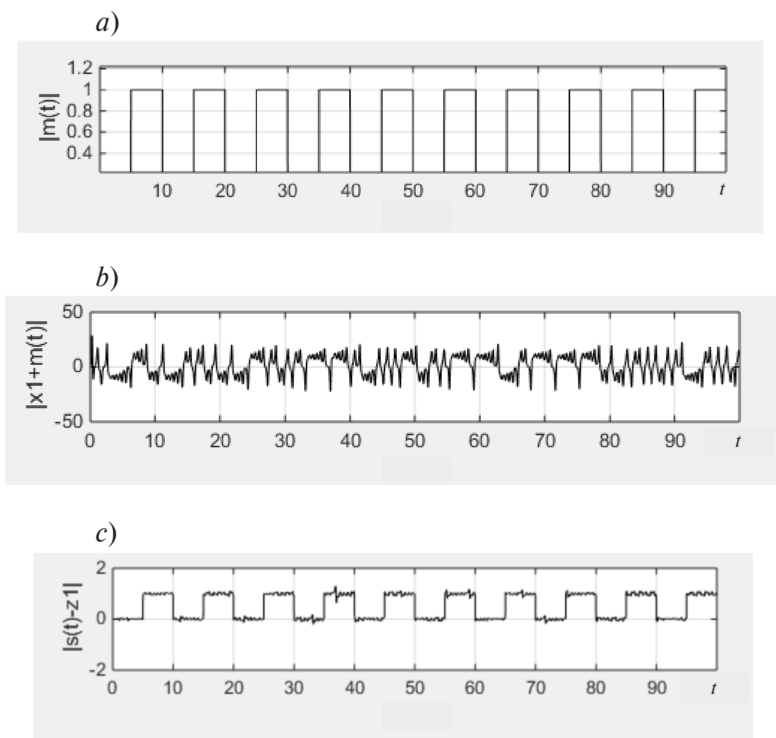

Fig. 6. Time diagrams in the transmission system: *a* – signal at the input of the system; *b* – signal in the communication line; *c* – signal at the output of the system

Conclusion

A control synthesis technique for complete synchronization of chaotic oscillations in a nonlinear system using phase vector feedback is proposed. The feedback coefficient ensures the equality of the spectra of Lyapunov characteristic exponents at different values of the parameters of chaotic generators.

The possibility of using the proposed method of feedback synthesis by the example of a system consisting of two Lorentz systems has been investigated. The results of computational experiments confirmed the complete synchronization of chaotic oscillations in the information transmission system with chaotic masking.

The advantage of the proposed method of forming a complete synchronization of two chaotic systems, which combines the introduction of the mutual influence of subsystems and control of the Lyapunov spectrum, lies in the possibility of transmitting a useful signal of lower amplitude, which could make the communication line stealthier. In addition, a full synchronization mode is provided with different parameters of the transmitter and receiver and mutual influence in a wider range.

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