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SYNTHESIS OF DECENTRALIZED ROBUST STABILIZING CONTROL FOR THE SYSTEMS WITH PARAMETRIC PERTURBATIONS

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The paper considers the problem of robust stabilization of large-scale systems with parametric perturbations within set number intervals. Design of systems with robust properties is among the most important problems of control theory. It allows to describe dynamics of the initial object by a mathematical model using a vector differential equation with interval coefficients. The paper states the problem of synthesis of stabilizing control with a pre-assigned degree of robust stability for closed systems. Scalar optimization function and Lyapunov—Razumikhin function were used to identify many stabilizing regulators. Parameters of robust control regulators are determined by the solutions of two Riccati equations: the first one corresponding to the nominal parameters of the object and the second — to variations of the object parameters. Sufficient conditions for robust stability of the closed system are obtained.

Keywords: robust control, large-scale systems, parametric perturbations, decentralized structure, Lyapunov—Razumikhin functions, pre-assigned degree of robust stability.

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СИНТЕЗ ДЕЦЕНТРАЛИЗОВАННОГО РОБАСТНОГО СТАБИЛИЗИРУЮЩЕГО УПРАВЛЕНИЯ СИСТЕМАМИ С ПАРАМЕТРИЧЕСКИМИ ВОЗМУЩЕНИЯМИ

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Рассмотрена задача робастной стабилизации крупномасштабных систем с параметрическими возмущениями, которые лежат внутри заданных числовых промежутков. Проектирование систем с робастными свойствами является одной из важнейших задач теории автоматического управления. Это позволяет описывать динамику исходного объекта с помощью векторно-матричных дифференциальных уравнений с интервальными коэффициентами. Для класса замкнутых систем сформулирована задача синтеза стабилизирующего управления с заданной степенью робастной устойчивости. С использованием скалярно-оптимизационных функций и функций Ляпунова—Разумихина выделяется множество стабилизирующих регуляторов. Параметры регуляторов определяются решениями уравнения Риккати, связанного с номинальными параметрами объекта, и уравнения Риккати, составленного для вариаций параметров объекта. Получены достаточные условия существования управления, обеспечивающего робастную стабилизацию исходной системы. Замкнутая система имеет заданную степень робастной устойчивости.

Ключевые слова: робастное управление, крупномасштабные системы, параметрические возмущения, децентрализованная структура, функции Ляпунова–Разумихина, устойчивость с заданной степенью робастности.

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Introduction

The defining property of a systems with inexact parameters is the presence of boundaries of a set of initial states and a set of possible trajectories. The systems of this class are of interest not only because of the plentitude of new mathematical problems, but also due to widespread applications of the theory of control of large-scale dynamic systems for the management of both technical objects and economic processes [1–6].

The history of the problem of robustness of the basic dynamic characteristics with respect to various perturbations goes back to the studies of Russian scientists A.A. Andronov and L.S. Pontryagin. To date, a wide range of methods has been developed providing invariance of the characteristics of systems with respect to parametric perturbations based on a variational approach (introducing feedback concerning sensitivity functions), the introduction of a loop with an infinitely large gain, and organization of sliding modes. However, they provide stability only under insignificant variations in parameters.

Modern approaches are focused on ensuring robust stability and quality under significant parametric perturbations. In most cases, these approaches use the robust control synthesis based on solving interval matrix equations of Lyapunov, Sylvester, and Riccati [7, 8, 10–15], or the approaches based on the theory of dynamic games using the strategies of guaranteed result [9, 16].

This paper considers the solution to a robust stabilization problem for systems with parametric perturbations, which are described by interval values of the matrix elements. The author develops an approach to the formation of sufficient conditions for the stability of continuous systems and the synthesis of stabilizing controls with a given measure of robustness, based on using the scalar optimization function of the set and the Lyapunov–Razumikhin interval function [16, 17].

The technique of synthesis of control with robust properties with respect to parametric perturbations uses two-sided target inequalities that determine the requirements for the upper and lower boundaries of the degree of stability of a closed system.

Formulation of the problem

Consider a mathematical model of a large-scale object with parametric perturbations in a form of a system of differential equations with interval coefficients

$$\dot{x} = \tilde{A}x + \tilde{B}u = \left(\tilde{A}_D + \tilde{A}_O\right)x + \tilde{B}u,\ x(0) = x_0, \tag{1}$$

where $x \in \mathbf{R}^n$ is a vector of phase coordinates; $u \in \mathbf{R}^m$ is a vector of control actions; $\tilde{A} \in \mathbf{IR}^{n\times n}$ is an interval matrix of system parameters, $\tilde{A} = \left(\tilde{a}_{ij}\right)_{i,j=1}^{n}$, $\tilde{a}_{ij} = \left[\underline{a}_{ij}; \overline{a}_{ij}\right] \in \mathbf{IR}$; $\tilde{A}_D$ = blockdiag $\left\{\tilde{A}_{ii}\right\}_{i=1}^{N}$ is a diagonal matrix with the blocks equal to the diagonal blocks of the matrix $\tilde{A}$; $\tilde{A}_O$ = block $\left\{\tilde{A}_{ij}\right\}_{i,j=1}^{N}$ is a non-diagonal matrix, the blocks of which are non-diagonal blocks of the matrix $\tilde{A}$; $\tilde{B} \in \mathbf{IR}^{m\times n}$ is a matrix of the control action transmission.

The requirements to the dynamic properties of the system are determined by a given degree of stability, which characterizes, on one hand, the location of the eigenvalues of the matrix of the closed system in the open left half-plane, and, on the other hand, the attenuation of some function characterizing the generalized distance from the integral curves of the system to the origin of the phase space coordinates. In this case, it is necessary to provide the required degree of stability in the presence of parametric perturbations.

The degree of stability of the interval system when changing the elements of the matrices $\tilde{A}$ and $\tilde{B}$ is defined by the interval number $\tilde{\alpha} = [\underline{\alpha}; \overline{\alpha}]$.

The index of robust stability is understood as a number e

$$e = 0{,}5 \text{wid}\, \tilde{\alpha} \,/\, \text{med}\, \tilde{\alpha},$$

characterizing the relative change in the degree of stability of the interval system. Here $\tilde{\alpha}$ is the width and $\tilde{\alpha}$ is the median of the interval number $\tilde{\alpha}$.

The synthesis of a stabilization system under conditions of parametric uncertainty consists in determining control in the form of feedback with respect to the state vector $u = U(x,t)$, which ensures the fulfillment of target conditions in the form of two-sided inequalities

$$\begin{aligned} \underline{M}\|x_0\|\exp\left[-\overline{\alpha}\left(t-t_0\right)\right] \le \left\|x\left(t, t_0, x_0, A, B\right)\right\| \le \\ \le \overline{M}\|x_0\|\left[-\underline{\alpha}\left(t-t_0\right)\right], \end{aligned} \tag{2}$$

determining the minimum and maximum rate of the phase coordinate decay in the transition mode for all possible variations of the parameters ($A \in \tilde{A}$, $B \in \tilde{B}$). Here $\exp\left[-\tilde{\alpha}\left(t-t_0\right)\right]$ is the interval function, which is a natural interval extension of the real function $\exp\left[-\alpha\left(t-t_0\right)\right]$.

$$\exp\left[-\tilde{\alpha}\left(t-t_0\right)\right] = \left[\exp\left[-\overline{\alpha}\left(t-t_0\right)\right]; \exp\left[-\underline{\alpha}\left(t-t_0\right)\right]\right].$$

Introduce the interval Lyapunov function

$$\tilde{V}\left(x\right) = x^T \tilde{P} x = \left[x^T \underline{P} x\,; x^T \overline{P} x\right], \tag{3}$$

which is a natural interval extension of the scalar Lyapunov function, and $\tilde{P} = \left[\underline{P}; \overline{P}\right] \in \mathbf{IR}^{n\times n}$ is a symmetric positive definite interval matrix (any $P \in \tilde{P}$ is a symmetric positive definite matrix). The Lyapunov function (3) allows reformulating the problem of ensuring dynamic stability (2) as the problem of determining control law with feedback that provides a given decay rate of the Lyapunov interval function on the trajectories of perturbed motion $\dot{\tilde{V}}\left(x\right) \le -2\tilde{\alpha}\tilde{V}\left(x\right)$,

$$\dot{\tilde{V}}\left(x\right) \le -2\tilde{\alpha}\tilde{V}\left(x\right), \tag{4}$$

where $\tilde{\alpha} = \left[\underline{\alpha}; \overline{\alpha}\right] \in \mathbf{IR}$ is the lower and upper boundaries of the required degree of stability of the closed system; $\dot{\tilde{V}}\left(x\right)$ is the derivative of the Lyapunov interval function (3), calculated by virtue of system (1), closed by the regulator $u = U(x,t)$.

Synthesis technique

The essence of the problem of the robust control synthesis lies in the non-uniqueness of the mapping given by the differentiation operator in the system of equations with an indefinitely set right-hand side. To each fixed point (x,θ) of the phase space and the number equal to the value of the positively defined function $V(x,\theta)$ at this point, there is a corresponding set of values of the functional $V(x_t)$ on the set of trajectories x_t coming to this point.

A condition for the effective application of the direct Lyapunov method to the problems of robust stability of the systems with interval coefficients is either the knowledge of the vortices $\{x_t\}$, the segments of the solutions of the system of differential equations that come to this point and correspond to the equivalent initial conditions satisfying the relation $\left\|x_t\left(t_0, x_0, A, B\right)\right\| \le \varepsilon$, or the possibility of constructing estimates based on using scalar optimization functions of the sets of specified vortices.

Using a centered representation of interval matrices $\tilde{A}$ and $\tilde{B}$, the system (1) is reduced to a form [16]

$$\dot{x} = \left(A_0 + \delta\tilde{A}\right)x + \left(B_0 + \delta\tilde{B}\right)u, \quad x(0) = x_0, \tag{5}$$

where $A_0 = \text{med}\,\tilde{A} \in \mathbf{R}^{n\times n}$, $B_0 = \text{med}\,\tilde{B} \in \mathbf{R}^{n\times m}$ are the matrices of median (nominal) values of the parameters of the studied system, whereas $\delta\tilde{A} = \left[-\text{rad}\,\tilde{A}; +\text{rad}\,\tilde{A}\right] \in \mathbf{IR}^{n\times n}$, $\delta\tilde{B} = \left[-\text{rad}\,\tilde{B}; + +\text{rad}\,\tilde{B}\right] \in \mathbf{IR}^{n\times m}$ are interval matrices characterizing the parameter variations.

Denote by $\mu(x(\theta))$ the vortex of integral lines with the vertex at the point (x,θ), which is the set of segments of integral lines of the system of differential equations of the perturbed motion (5), corresponding to the condition $\mu\left(x(\theta)\right) = \left\{x_t \middle| V\left(x(\theta)\right) \le r(L), \theta \ge T\right\}$.

Let us introduce a scalar optimization function $R(x)$ of the set $\mu(x(\theta))$, which establishes a correspondence between the set $\mu(x(\theta))$ and the points of the number axis $\mathbf{R}^1_+$

$$R(x) = \sup\left\{\dot{V}(x) \middle| x(t) \in \mu\left(x(\theta)\right)\right\}. \tag{6}$$

Thus, the scalar optimization function $R(x)$ is determined by the maximum value of the functional $V(x_t)$ on a bounded set of integral curves, along which function $V(x)$ decreases [18].

The condition of exponential stability (4) of the system with parametric perturbations (5) at the scalar optimization function $R(x)$ (6) has the form

$$R(x) \le -\tilde{\alpha} V(x) \tag{7}$$

in the domain

$$\Theta = \left\{(t,x) \middle| t \ge T \ge t_0 + l,\right.$$

$$\left. 0 \le V(x) \le r(L) = \sup\left\{V(x) \middle| \|x\| \le L\right\}\right\}.$$

Let us construct for the function $\tilde{V}(x)$ a centered form corresponding to the adopted mathematical model of the system with parametric perturbations

$$\tilde{V}(x) = V_0 + \delta\tilde{V} = x^T P_0 x + x^T \delta\tilde{P} x = x^T\left(P_0 + \delta\tilde{P}\right)x, \tag{8}$$

where $P_0 = \text{med}\ \tilde{P} \in \mathbf{R}^{n\times n}$ is the matrix of median values of the quadratic form; $\delta\tilde{P} = \left[-\text{rad}\ \tilde{P}; +\text{rad}\ \tilde{P}\right] = \left[-1;+1\right] \text{rad}\ \tilde{P} \in \mathbf{IR}^{n\times n}$ is a matrix related to parameter variations.

Now calculate the total derivative of the Lyapunov function (8) by virtue of the system with parametric perturbations (5)

$$\dot{\tilde{V}}(x) = \left(\partial\tilde{V}(x)/\partial x\right)^T (dx/dt) =$$
$$= \dot{x}^T\left(P_0 + \delta\tilde{P}\right)x + x^T\left(P_0 + \delta\tilde{P}\right)\dot{x} \subseteq$$
$$\subseteq x^T\left[\left(A_0 + \delta\tilde{A}\right)^T\left(P_0 + \delta\tilde{P}\right) + \left(P_0 + \delta\tilde{P}\right)\left(A_0 + \delta\tilde{A}\right)\right]x +$$
$$+2x^T\left(P_0 + \delta\tilde{P}\right)\left(B_0 + \delta\tilde{B}\right)u.$$

Then the condition of exponential stability (7) for system (5) is determined by the inequality

$$\begin{aligned} R(x) = \sup\Big\{&x^T\left[\left(A_0 + \delta\tilde{A}\right)^T\left(P_0 + \delta\tilde{P}\right)\right]x + \\ &+x^T\left[\left(P_0 + \delta\tilde{P}\right)\left(A_0 + \delta\tilde{A}\right)\right]x + \\ &+2x^T\left(P_0 + \delta\tilde{P}\right)\left(B_0 + \delta\tilde{B}\right)u \,\Big|\, x \in \mu\left(x(\theta)\right)\Big\} \le \\ &\le -(\alpha_0 + \delta\tilde{\alpha})\, x^T\left(P_0 + \delta\tilde{P}\right)x, \end{aligned} \tag{9}$$

where $\alpha_0 = \text{med}\ \tilde{\alpha} \in \mathbf{R}^1$, $\delta\tilde{\alpha} = \left[-1;+1\right] \text{rad}\ \tilde{\alpha}$.

The set of all stabilizing controls that satisfy inequality (9) is described by the relation

$$\begin{aligned} u = \Bigg\{&-\frac{\left(B_0 + \delta\tilde{B}\right)^T\left(P_0 + \delta\tilde{P}\right)x}{2\left\|x^T\left(P_0 + \delta\tilde{P}\right)\left(B_0 + \delta\tilde{B}\right)\right\|^2} \times \\ &\times\left[x^T\left[2\left(A_0 + \delta\tilde{A}\right)\left(P_0 + \delta\tilde{P}\right)\right]x + x^T\left[\left(\alpha_0 + \delta\tilde{\alpha}\right)\left(P_0 + \delta\tilde{P}\right)\right]x\right]\Bigg\}. \end{aligned} \tag{10}$$

In relation (10), the matrix P_0 satisfies the Riccati matrix equation

$$A_{0\alpha}^T P_0 + P_0 A_{0\alpha} - P_0 F_0 P_0 + Q_0 = 0. \tag{11}$$

Equation (11) corresponds to the nominal system, and the matrices have a form $A_{0\alpha} = A_0 + 0{,}5\alpha_0 I_n$, $F_0 = 10 B_0 B_0^T + 2I_n$, $Q_0 = 2A_0^T A_0$.

The interval matrix $\delta\tilde{P}$

$$\begin{aligned} \delta\tilde{P} = \Big\{\delta P \in \mathbf{R}^{n\times n} \,\Big|\, &\delta A_\alpha^T \delta P + \delta P \delta A_\alpha - \delta P D \delta P + G = 0, \\ \Big|\, &\delta A_\alpha \in \delta\tilde{A}_\alpha,\ D \in \tilde{D},\ G \in \tilde{G}\Big\} \end{aligned}$$

defines the combined set of solutions of the Riccati interval matrix equation

$$\delta\tilde{A}_{\alpha}^{T}\delta\tilde{P}+\delta\tilde{P}\delta\tilde{A}_{\alpha}-\delta\tilde{P}\tilde{D}\delta\tilde{P}+\tilde{G}=0. \tag{12}$$

Here $\delta\tilde{A}_{\alpha}=\delta\tilde{A}+0,5\tilde{\alpha}I_{n}=\left[-1;+1\right]\left(\text{rad }\tilde{A}+0,5\text{ rad }\tilde{\alpha}I_{n}\right)\in\mathbf{IR}^{n\times n}$, $\tilde{D}=(10B_{0}B_{0}^{T}+4\delta\tilde{B}\delta\tilde{B}^{T}-$ $-2I_{n})\in\mathbf{IR}^{n\times n}$, $\tilde{G}=(\delta\tilde{A}^{T}\delta\tilde{A}-4P_{0}\delta\tilde{B}\delta\tilde{B}^{T}P_{0})\in\mathbf{IR}^{n\times n}$.

Equation (12) corresponds to variations in the parameters of system (1).

The set of regulators providing robust stability of the interval system (5) with a given indicator e is determined by the following expression

$$u\left(x\right)=-B_{0}^{T}\left(P_{0}+\delta\tilde{P}\right)x=-\left(K_{0}+\delta\tilde{K}\right)x, \tag{13a}$$

where $K_{0}=B_{0}^{T}P_{0}\in\mathbf{R}^{m\times n}$, $\delta\tilde{K}=B_{0}^{T}\delta\tilde{P}\in\mathbf{IR}^{m\times n}$ or

$$U\left(x\right)=\tilde{K}x=\left(\text{med }\tilde{K}\pm0,5\text{wid }\tilde{K}\right)x, \tag{13b}$$

where $\tilde{K}=-\tilde{B}^{T}\tilde{P}$ is the matrix of interval feedback coefficients; med $\tilde{K}=\left(\overline{K}+\overline{K}\right)/2$ is the median of the interval matrix $\tilde{K}$, setting nominal values of regulator parameters; wid $\tilde{K}=\left(\overline{K}-\underline{K}\right)$ is the width of the matrix $\tilde{K}$, which determines the maximum margin of the parameters.

Control (13b) is with the matrix $\tilde{P}$, which is a combined set of solutions

$$\begin{gathered}\tilde{P}=\left\{P\in\mathrm{R}^{n\times n}\,\middle|\,P\left(A+\alpha E_{n}\right)+\left(A+\alpha E_{n}\right)^{T}P-\right.\\ \left.-2PBB^{T}P=0\,,A\in\tilde{A},\,B\in\tilde{B},\,\alpha\in\tilde{\alpha}\right\}.\end{gathered} \tag{14}$$

The matrix $\tilde{P}$ defined by expression (14) satisfies the Riccati interval matrix equation and can be found by the technique presented in [19],

$$\tilde{P}\left(\tilde{A}+\tilde{\alpha}E_{n}\right)+\left(\tilde{A}+\tilde{\alpha}E_{n}\right)^{T}\tilde{P}-2\tilde{P}\tilde{B}\tilde{B}^{T}\tilde{P}=0. \tag{15}$$

Control (13b) provides robust stabilization of the system $\dot{x}=\left(\tilde{A}-\tilde{B}\tilde{K}\right)x$ with a degree of stability $\tilde{\alpha}$ belonging to the given interval $\left[\underline{\alpha};\overline{\alpha}\right]$.

The robust regulator (13) has a centralized structure, which requires significant computational costs to determine its parameters due to the need of solving the Riccati interval equation of a sufficiently large dimension. Besides, the regulator is difficult to implement, since it requires information on the state of the entire system to form a control action.

The synthesis of decentralized robust control

To reduce the costs of design and implementation, we will seek a robust regulator in a class of decentralized structures

$$\begin{gathered}\tilde{K}=\left\{K=\text{blockdiag}\left\{K_{ii}\right\}_{i=1}^{n}\in\mathrm{R}^{m\times n}\right|\\ \left|\operatorname{Re}\lambda_{i}\left(A-BK\right)<0,\,i=\overline{1,n},A\in\tilde{A},B\in\tilde{B}\right\}.\end{gathered} \tag{16}$$

To this end, using the idea of inverse optimization, we carry out a decomposition of the Riccati equation (15), presenting the matrix $\tilde{A}$ as a sum of the block diagonal matrix $\tilde{A}_D$ and the block non-diagonal matrix $\tilde{A}_O$

$$\tilde{P}\left(\tilde{A}_D + \tilde{\alpha}E_n\right) + \left(\tilde{A}_D + \tilde{\alpha}E_n\right)^T \tilde{P} - \\ -2\tilde{P}\tilde{B}\tilde{B}^T\tilde{P} + \tilde{P}\tilde{A}_O + \tilde{A}_O^T\tilde{P} = 0. \quad (17)$$

To satisfy structural constraints (16), we introduce into the Riccati equation (17) an additional term $\tilde{Q}_O$, the value of which is determined by the intensity of connections between the subsystems

$$\tilde{Q}_O = -\left(\tilde{A}_O^T\tilde{P} + \tilde{P}\tilde{A}_O\right). \quad (18)$$

Then it becomes possible to decompose equation (17) into N equations, each of which corresponds to the i^{th} subsystem included in the large-scale system (1)

$$\tilde{P}_{ii}\left(\tilde{A}_{ii} + \tilde{\alpha}E_{n_i}\right) + \left(\tilde{A}_{ii} + \tilde{\alpha}E_{n_i}\right)^T \tilde{P}_{ii} - \\ -2\tilde{P}_{ii}\tilde{B}_{ii}\tilde{B}_{ii}^T\tilde{P}_{ii} = 0,\ i = \overline{1,N}, \quad (19)$$

whereas matrix $\tilde{P}$ acquires the desired diagonal structure $\tilde{P} =$ blockdiag $\left\{\tilde{P}_{ii}\right\}_{i=1}^{N}$. The block elements of the matrix $\tilde{Q}_O$ in relation (18) are determined by the following formula

$$\tilde{Q}_O = \text{block}\left\{\tilde{Q}_{ij}\right\}_{i,j=1}^{N} = \begin{cases} \tilde{Q}_{ii} = 0, i = j \\ \tilde{Q}_{ij} = \sum_{i=1}^{N}\left(\tilde{P}_{ii}\tilde{A}_{ij} + \tilde{A}_{ji}^T\tilde{P}_{jj}\right), i \neq j \end{cases}. \quad (20)$$

The matrix $\tilde{K}$ of feedback coefficients, due to the block diagonal structure of the matrices $\tilde{B}$ and $\tilde{P}$, is also a block diagonal matrix, and the decentralized robust regulator takes the form

$$u_i\left(x_i\right) = \tilde{K}_{ii}x_i = -\tilde{B}_{ii}^T\tilde{P}_{ii}x_i, i = \overline{1,N}, \quad (21)$$

where the interval matrix

$$\tilde{P}_{ii} = \left\{P_{ii} \in \mathrm{R}^{n_i \times n_i} \,\middle|\, P_{ii}\left(A_{ii} + \alpha E_{n_i}\right) + \left(A_{ii} + \alpha E_{n_i}\right)^T P_{ii} - \right. \\ \left. -2P_{ii}B_{ii}B_{ii}^T P_{ii} = 0,\ A_{ii} \in \tilde{A}_{ii},\ B_{ii} \in \tilde{B}_{ii},\ \alpha \in \tilde{\alpha}\right\}$$

is a combined set of solutions of the Riccati interval equation (19).

Relation (21) defines the set of stabilizing regulators. Any regulator with the feedback coefficient $K_{ii} \in \tilde{K}_{ii}$ ensures stability of interconnected subsystems

$$\dot{x}_i = \left(\tilde{A}_{ii} + \tilde{B}_{ii}\tilde{K}_{ii}\right)x_i + \sum_{j=1, j\neq i}^{N}\tilde{A}_{ij}x_j, i = \overline{1,N} \quad (22a)$$

or stability of the large-scale system as a whole

$$\dot{x} = \left(\tilde{A}_D + \tilde{B}\tilde{K}\right)x + \tilde{A}_O x. \tag{22b}$$

Also, as in the case of a centralized regulator, it is expedient to choose the feedback coefficient K_{ii} in the following way

$$K_{ii} = \left(\text{med}\,\tilde{K}_{ii} \pm 0{,}5\text{wid}\,\tilde{K}_{ii}\right),\ i = \overline{1,N}, \tag{23}$$

thereby determining the nominal parameters of the decentralized regulator $K_{0ii} = \text{med}\,\tilde{K}_{ii} = \left(\overline{K}_{ii} + \underline{K}_{ii}\right)/$ /2 and the margin for their change $\delta\text{K}_{\text{ii}} = \text{wid}\,\tilde{K}_{ii} = \left(\overline{K}_{ii} - \underline{K}_{ii}\right)$.

Properties of the decentralized control

The decentralized control is built in a form of feedback on the phase vector taking into account the mutual influence of subsystems, and ensures the degree of stability of the closed-loop system within a given interval. The use of decentralized control makes it possible to reduce the synthesis costs due to the decomposition of the Riccati matrix equations, which are solved during the synthesis associated with subsystems whose dimensions are less than the dimension of the original large-scale system. The proof of the theorem on the properties of decentralized control is based on a scalar optimization function, which is an analogue of the derivative of the Lyapunov function for systems with parametric perturbations.

The properties of a large-scale system closed by a decentralized regulator are defined in the following theorem.

Theorem. Suppose that for each subsystem included in a large-scale system, the following conditions are fulfilled:

1) rank control criterion

$$\text{rank}\,\tilde{D}_{ii} = n_i,\ \tilde{D}_{ii} = \left(\tilde{B}_{ii}\middle|\tilde{A}_{ii}\tilde{B}_{ii}\middle|\ldots\middle|\tilde{A}_{ii}^{(n_i-1)}\,\tilde{B}_{ii}\right), \tag{24}$$

2) existence of the inverse matrix for the test matrix of the form

$$\tilde{W}_i = \left(\left(\tilde{A}_{ii} + \tilde{\alpha}E_{n_i}\right)^T \otimes E_{n_i} + E_{n_i} \otimes \left(\tilde{A}_{ii} + \tilde{\alpha}E_{n_i}\right)\right), \tag{25}$$

where the rank of the test matrix equals n_i^2.

Then the decentralized control (21), including the regulator of the form (23), ensures the stability of the system (22) for all variations of the parameters satisfying conditions (25) and (26), while the degree of stability of the closed system belongs to the interval $\tilde{\alpha} = \left[\underline{\alpha};\overline{\alpha}\right]$.

Proof of the theorem. The fulfillment of the conditions of the theorem guarantees the existence of an external interval solution $\tilde{P}_{Bii} = \left[\underline{P}_{Bii};\overline{P}_{Bii}\right]$ of the Riccati equation (19), which includes the combined set of solutions $\tilde{P}_{ii} = \left[\underline{P}_{ii};\overline{P}_{ii}\right]$.

Then there exists an interval quadratic form

$$\tilde{V}_i\left(x_i\right) = x_i^T\tilde{P}_{Bii}x_i = \left[x_i^T\underline{P}_{Bii}x_i\,;x_i^T\overline{P}_{Bii}x_{ii}\right], \tag{26}$$

satisfying two-sided inequality

$$\underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right)\|x\|^2 \le \tilde{V}_i\left(x_i\right) \le \overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right)\|x\|^2, \quad (27)$$

where $\underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right)$ is the lower boundary of the minimum eigenvalue, while $\overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right)$ is the upper boundary of the maximum eigenvalue of the interval matrix $\tilde{P}_{Bii}$, which are calculated by the formulas

$$\tilde{\lambda}_{\min}\left(\tilde{P}_{Bii}\right) = \left[\lambda_{\min}\left\{\text{med } \tilde{P}_{Bii}\right\} - \varepsilon_{\min}; \lambda_{\min}\left\{\text{med } \tilde{P}_{Bii}\right\} + \varepsilon_{\min}\right], \quad (28)$$

$$\tilde{\lambda}_{\max}\left(\tilde{P}_{Bii}\right) = \left[\lambda_{\max}\left\{\text{med } \tilde{P}_{Bii}\right\} - \varepsilon_{\max}; \lambda_{\max}\left\{\text{med } \tilde{P}_{Bii}\right\} + \varepsilon_{\max}\right]. \quad (29)$$

Here the residual vector ε in (28) and (29) is calculated by the formulas

$$\varepsilon_{\min} = \|\ |\xi_{\min}|\ \|_2, \varepsilon_{\max} = \|\ |\xi_{\max}|\ \|_2, \|\ |\xi|\ \|_2 = \left(\sum_1^n |\xi_i|^2\right)^{1/2},$$

$$|\xi| = \text{col}|\xi_i|, |\tilde{\xi}_i| = \max\left\{|\underline{\xi}_i|, |\overline{\xi}_i|\right\}.$$

Therefore, the quadratic form (26) is a Lyapunov function of the interval type for system (1) with parametric perturbations. Let us calculate the total derivative of the Lyapunov interval function

$$\tilde{V}(x) = \sum_{i=1}^N \tilde{V}_i\left(x_i\right) = x^T \tilde{P}_B x, \tilde{P}_B = \text{blockdiag}\left\{\tilde{P}_{Bii}\right\}_{i=1}^N$$

due to the system, closed by the decentralized control (21)

$$\dot{\tilde{V}}(x) = 2x^T \tilde{P}_B\left(\tilde{A}_D - \tilde{B}\tilde{B}^T \tilde{P}_B + \tilde{A}_O\right)x \subseteq$$

$$\subseteq x^T\left(\tilde{P}_B \tilde{A}_D + \tilde{A}_D^T \tilde{P}_B - 2\tilde{P}_B \tilde{B}\tilde{B}^T \tilde{P}_B + \tilde{P}_B \tilde{A}_O + \tilde{A}_O^T \tilde{P}_B\right)x.$$

Matrices $\tilde{P}_{Bii}$ are solutions to the matrix Riccati equations (19), which are, in total, equivalent to equation (17) when choosing the matrix $\tilde{Q}_O$ in accordance with (20). Therefore, for the derivative of the Lyapunov function under consideration, the following inequality holds:

$$\dot{\tilde{V}}(x) \le -2\tilde{\alpha} x^T\left(\tilde{P}_B + \tilde{Q}_O\right)x. \quad (30)$$

The relation (30) yields the fulfillment of the inequality

$$\tilde{V}\left(x_0\right)\exp\left[-2\overline{\alpha}\left(t - t_0\right)\right] \le \tilde{V}(x) \le \tilde{V}\left(x_0\right)\exp\left[-2\underline{\alpha}\left(t - t_0\right)\right], \quad (31)$$

which testifies to the exponential stability with respect to function $\tilde{V}(x)$. Using estimate (27), we replace $\tilde{V}\left(x_0\right)$ in the right-hand side of the last inequality with the larger quantity $\overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right)\|x_0\|^2$, and $\tilde{V}(x)$, with the smaller quantity $\underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right)\|x\|^2$; then for solutions of system (22) with the synthesized decentralized regulator, there holds the estimate from above

$$\left\|x\left(t,\, t_0,\, x_0,\, A, B\right)\right\| \leq$$
$$\leq \left(\overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right) / \underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right)\right)^{1/2} \left\|x_0\right\| \exp\left[-\underline{\alpha}\left(t-t_0\right)\right]. \tag{32}$$

Similarly, replacing the quantity $\tilde{V}\left(x_0\right)$ in the left-hand side of inequality (31) with the smaller quantity $\underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right)\left\|x_0\right\|^2$, and $\tilde{V}\left(x\right)$, with the larger quantity $\overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right)\left\|x\right\|^2$, we obtain the following estimate from below for the solution of the system

$$\underline{\lambda}_{\min}\left(\tilde{P}_{Bii}\right) / \left(\overline{\lambda}_{\max}\left(\tilde{P}_{Bii}\right)\right)^{1/2} \left\|x_0\right\| \exp\left[-\overline{\alpha}\left(t-t_0\right)\right] \leq \left\|x\left(t, t_0, x_0, A, B\right)\right\|. \tag{33}$$

Inequalities (32) and (33) indicate the fulfillment of the target condition (2). The solutions of system (1) with any regulator from the family of interval regulators (21), including regulator (23), are exponentially stable. The obtained inequalities for the solutions of system (1), (22) are valid for all variations of the parameters satisfying the conditions of the theorem; therefore, the closed system has the property of robust stability with respect to parametric perturbations.

Conclusion

The considered technique for the synthesis of decentralized control of a large-scale system ensures the stability of perturbed motions in the conditions of the interval setting of subsystem parameters. The parameters of the stabilizing regulator are calculated using the solution of the Riccati interval matrix algebraic equation. The decomposition of the matrix equation, based on the ideas of inverse optimization, allows reducing the design costs using a decentralized structure of the regulator.

The proposed procedure for the synthesis of decentralized control provides stability of a large-scale system in comparison with other methods of decentralized control, taking into account the mutual influence of subsystems. The use of the interval model of the system ensures robust stability with respect to internal parametric disturbances, in contrast to control based on Hardy spaces (H_2, H_∞ – control methods). In comparison with the methods based on V.L. Kharitonov's theorems, the considered method allows not only the synthesis of systems with interval parameters, but also the synthesis of systems with the required properties.

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