

# Mathematical Modelling: Methods, Algorithms, Technologies

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## **ON FRACTAL, STATISTICAL AND MORPHOLOGICAL METHODS OF DIGITAL IMAGE ANALYSIS IN MEDICAL RESEARCH**

The applicability of some methods of analysis and classification to the images of biomedical preparations is discussed. For images of blood, connective tissue and bone tissue fractal characteristics were calculated. When researching the images of blood obtained by the sensitive crystallization method, statistical, morphological and spectral signs were obtained, which were classified by a software modeled artificial neural network. The results of numerical experiments are given.

IMAGE ANALYSIS; FRACTAL METHODS; NEURAL NETWORKS; BIOMEDICAL PREPARATIONS.

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## **О ФРАКТАЛЬНЫХ, СТАТИСТИЧЕСКИХ И МОРФОЛОГИЧЕСКИХ МЕТОДАХ АНАЛИЗА ЦИФРОВЫХ ИЗОБРАЖЕНИЙ В МЕДИЦИНСКИХ ИССЛЕДОВАНИЯХ**

Статья посвящена исследованию применимости методов анализа и классификации к изображениям биомедицинских препаратов: жидкой крови, соединительной и костной тканей. Для этих классов изображений вычислены фрактальные характеристики. Также исследованы изображения препаратов крови и экстрактов растений, полученных методом чувствительной кристаллизации. Для этих классов вычислены статистические, морфологические и спектральные признаки, которые затем классифицированы с помощью программно-моделируемой искусственной нейронной сети. Приведены результаты экспериментов.

АНАЛИЗ ИЗОБРАЖЕНИЙ; ФРАКТАЛЬНЫЕ МЕТОДЫ; НЕЙРОННАЯ СЕТЬ; БИОМЕДИЦИНСКИЕ ПРЕПАРАТЫ.

In biology and medicine the possibility of automatic processing large sets of images, determining the type of issue analyzed, revealing tumors and detecting some foreign compositions may have an influence on the research and to assist a physician to form a diagnosis [1].

For complex biological systems we as a rule do not have adequate mathematical models, but one can register the results of their functioning and obtain digital images. Such an approach is quite appropriate when studying the action of ultralow doses of a medicine on a biological system: since it is difficult to describe the

results of the effect by using only one main factor (sign), we register the states of a vital functions sign, for example blood. Digital images of the registered states may be analyzed by mathematical methods. Hence, such an image may be considered as the phase portrait of the system studied at a point of time.

As a rule such digital images have rather complex structure and to analyze it the methods of fractal analysis are applied. The computation of fractal dimensions allows us to analyze and classify images of pharmacological preparations, living organism tissue, geological band fractures. In practice the images are unions of different

fractals (so called multifractals), being every fractal has its own fractal dimension and all the fractal sets are interpenetrative. Hence the methods of multifractal analysis were worked out that allows obtaining a multifractal spectrum – a set of fractal dimensions.

The results of the research of different classes of biomedical preparations that is based on the using fractal methods is given in [2]. To compute the fractal dimension of an image (the Minkovsky dimension) the modified fractal signature method was used.

The concept of fractal signature and the method were introduced in [3] to analyze and classify textures. The method is based on the Mandelbrot idea about the approximate calculation of the length of coastline by measuring the area of a strip that contains the line and has width  $2\delta$ , where  $\delta$  is a fixed number. Then the length is approximately the area divided by  $2\delta$ . It should be noted that the length increases as  $\delta$  decreases. At the same time Mandelbrot noted that there is an interval for  $\delta$  in which the value  $L(\delta)$  becomes stable.

The authors applied this method to measure the area of a gray level surface constructed by a digital image. The sequence of special blankets is constructed over the surface. For every blanket its volume is calculated, the surface area and the so called «fractal signature» (the ratio of the logarithm of the surface area to the logarithm of the scale) is defined. (It should be noted that the Minkovsky dimension of the area may be easily obtained from the fractal signature.) So we have a sequence of areas and signatures in accordance with the number of blankets. For two images we can compare the obtained signature vectors: the closeness between vectors shows an adjacency of their textures. In [4] this method was applied to the analysis of images of bone tissue for different color scales.

In [5] the authors applied the method to calculate the fractal dimension of a document image and called it the «modified fractal signature method». They used only two consecutive blankets and marked out the areas corresponding background, text and graphics in accordance with the Minkovsky dimension. In such a variant the method demonstrated high reliability and was successfully used in [2] to classify biomedical preparation images. The

image was divided into small boxes and the fractal dimension of both the area and boxes was calculated. That allowed us to obtain a «map» of the area and the dependence the surface area from the box size. The graphic of such a dependence was considered as a classification sign.

It is interesting to note that the method gave good results when classifying (defining a focusing degree) SAR (Synthetic Aperture Radar) images [6, 7].

As numerous researches show, the systems with chaotic behavior are very common in nature. Since in such a system one can trace a trajectory only on short time intervals, we should use an appropriate statistical description of the system asymptotic. It is the so called multifractal formalism that helps in deciding the problem. This technique is based on self-similarity property for an image, which gives a possibility to obtain the distribution of a probability measure. Such an approach allows us to associate the fractal properties of the image with the range of nonuniformity of the measure distribution. In the application to digital images to obtain such a distribution means to part the set of points of an image into subsets in which points have close characteristics. For a point one can use density defined as the limit (when the box size tends to zero) of the ratio of the logarithm of the box (containing the point) measure to the logarithm of the box side length. Thereby the image is a union of subsets containing the points with close densities. For each subset its fractal dimension is calculated. The set of these dimensions is called multifractal spectrum, which may be considered as the image characteristic. Numerical results showed that the multifractal spectrum method may be successfully applied to classify both bone tissues (health and affected by osteoporosis) and some classes of histological preparations [8].

It is reasonable that fractal and multifractal methods are applicable to definite classes of images. In medical diagnostic the images of preparations of blood and plant extracts obtained by sensitive crystallization by Pfaiffer method (the addition of a small dose of blood or plant extract to the solution of cuprum chloride) [9] are very important. It is well known that structure peculiarities of obtained

blood crystals allow revealing both clinical presentations of various diseases and existence of definite tendencies to their progress. For such images fractal methods are not quite suitable. When analyzing the described images we construct the following feature vectors:

- statistical textural ones obtained by using gray level cooccurrence matrix;
- morphological ones (skeleton representation);
- spectral ones (Gabor filters) [10].

Then these vectors were classified by a software modeled artificial neural network. To learn the network the RProp (Resilient Propagation) method was applied [11].

### The Methods of Solution

**The modified fractal signature method.** In what follows we use the terminology of [12]. Let  $F$  be a nonempty bounded subset of  $R^n$ ,  $\Omega$  – a finite  $\varepsilon$  – covering of  $F$  and  $N_\varepsilon(F) = |\Omega|$ . Then the following number is called capacity dimension of  $F$ :

$$\dim_B F = \lim_{\varepsilon \rightarrow 0} \frac{\log_2 N_\varepsilon(F)}{-\log_2 \varepsilon}. \quad (1)$$

This dimension is a kind of so called box-computing (or box-counting) dimensions (the name follows from the definition). In practice the formula (1) is rarely applied because to calculate  $N_\varepsilon(F)$  we have to save data about all the elements of the covering, whereas  $F$  may occupy only a part of it. Hence it is preferable to calculate the dimension that really is a box-counting one, but may be obtained by a more economic method.

We define  $\delta$ -parallel body for  $F$  (denoted by  $F_\delta$ ) as the set of points which are on the distance no greater than  $\delta$  from  $F$ :

$$F_\delta = \{x \in R^n : |x - y| \leq \delta, y \in F\}. \quad (2)$$

Denote  $n$ -dimensional volume of  $F_\delta$  by  $\text{Vol}^n(F_\delta)$ . If for a constant  $D$  (when  $\delta \rightarrow 0$ ) the limit of the ratio  $\text{Vol}^n(F_\delta) / \delta^{n-D}$  is positive and bounded, then  $D$  is called the Minkovsky dimension of the set  $F$  and denoted by  $\dim_M F$ . It is known [12] that the following relation holds (for nonempty bounded sets in  $R^n$ ):  $\dim_B F = \dim_M F$ .

Now we consider  $F = \{F_{ij}, i = 0, 1, \dots, K, j = 0, 1, \dots, L\}$  – the presentation of a gray

scale image, where  $F_{ij}$  is the intensity of the pixel with the coordinates  $(i, j)$ . We redefine  $F$  in a point with real coordinates  $(x, y)$ , where  $i < x < i + 1, j < y < j + 1$  by the corresponding value  $F_{ij}$ . The function  $F$  specifies a surface in 3-dimensional space and is called the surface of gray level function. For brevity this surface is also denoted by  $F$ . To calculate fractal dimension of this surface the so called «blanket technique» is used. Construct for the surface  $F$  the  $\delta$ -parallel body (blanket) with the help of the defined below top surface of blanket  $u_\delta(i, j)$  and bottom surface  $b_\delta(i, j)$ . Set  $u_0(i, j) = b_0(i, j) = F_{ij}$  and define surfaces for  $\delta = 1, 2, \dots$  by the following recurrent relation:

$$u_\delta(i, j) = \max\{u_{\delta-1}(i, j) + 1, \max_{|(m,n)-(i,j)| \leq 1} u_{\delta-1}(m, n)\}, \quad (3)$$

$$b_\delta(i, j) = \min\{b_{\delta-1}(i, j) - 1, \min_{|(m,n)-(i,j)| \leq 1} b_{\delta-1}(m, n)\}. \quad (4)$$

A point  $M(x, y)$  is included in the  $\delta$ -parallel body, if  $b_\delta(i, j) < M(x, y) < u_\delta(i, j)$ . It should be noted that in accordance with (3) and (4) the  $\delta$ -parallel body for a given  $\delta$  includes  $(\delta - 1)$ -parallel body. The volume of the  $\delta$ -parallel body is computed from  $u_\delta$  and  $b_\delta$ :

$$\text{Vol}_\delta = \sum(u_\delta(i, j) - b_\delta(i, j)). \quad (5)$$

The area  $A_\delta$  of the fractal surface (the surface of the gray level function) is computed as

$$A_\delta = \frac{\text{Vol}_\delta - \text{Vol}_{\delta-1}}{2}, \quad (6)$$

and the fractal dimension of the surface (the Minkovsky dimension) is calculated by the formula

$$D \approx 2 - \frac{\log_2 A_\delta}{\log_2 \delta}. \quad (7)$$

As it was shown in [15], to estimate  $D$  one may use only two values of  $\delta$ , namely  $\delta_1$  and  $\delta_2$  and the formula (7) is equivalent to the following relation:

$$D \approx 2 - \frac{\log_2 A_{\delta_1} - \log_2 A_{\delta_2}}{\log_2 \delta_1 - \log_2 \delta_2}. \quad (8)$$

When implementing the method we compute the area of the surface, areas of cells,

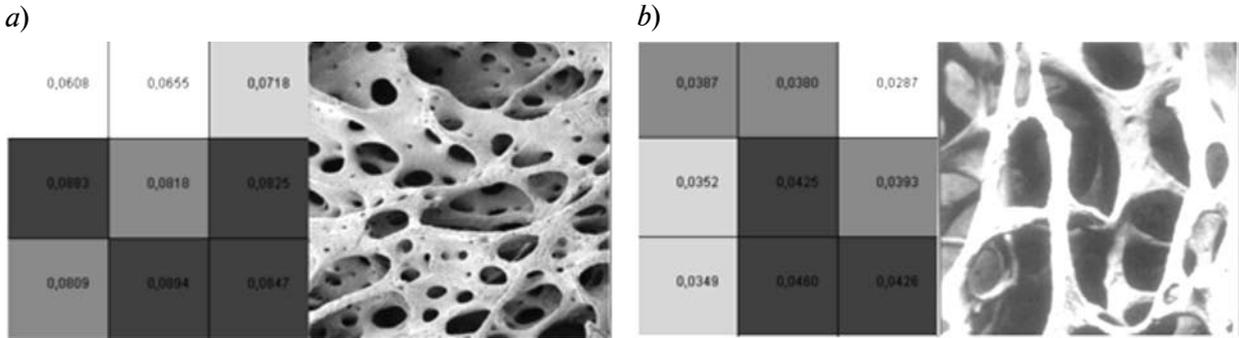


Fig. 1. Health bone tissue – *a*; affected bone tissue (osteoporosis) – *b*  
 The source image (right) and its map (left). The image size is 270×270, cell size is 90×90

fractal dimension for the surface and fractal dimensions of cells. We also construct a «map» of the image, where for a given cell size the cell areas were shown in a color scale. On Fig. 1 such maps are obtained for the cell size 90×90 (image size 270×270). The dependence both the surface area and the fractal dimension on cell size is shown on Fig. 2. We use such graphics as main features to compare the images of different classes. Generally speaking, it is sufficient to obtain both areas and dimensions only for the whole surface, but similar characteristics for the parts of the image give more obvious form to the results. It is the different fractal dimension values for different parts of an image that allowed the authors of [5] to distinguish text, graphics and background in text documents. In our experiments the method demonstrated good separability of features for all the classes of the preparations classified.

**Computing multifractal spectrum.** As it was mentioned above, we may consider an image

as the support of some distributed measure. Cover this support by nonintersecting boxes (cells)  $\{M_i\}$  (with intersections on the box boundaries) with the side length  $l$  and suppose that the box measure is  $l^{\alpha_i}$ , where  $\alpha_i$  are real numbers. Then one may define the point sets  $E_\alpha$  (for a given  $\alpha$ ) such that  $\alpha_i$  are close to  $\alpha$ . The set  $\{\alpha_i\}$  is a set of «densities» of the image points. For each point its density is computed through the box measure and the box side length [13]. Hence we collect the points with similar densities into the sets  $E_\alpha$ . For all such sets we compute their fractal dimensions  $f(\alpha_i)$ . By this means the image is considered as the union of interwoven subsets, being each of them has own fractal dimension. The set of these dimensions forms multifractal spectrum (MFS).

We note that locating of sets  $E_\alpha$  means some categorization of the image points, and the defined measure describes this categorization (for digital images measure is naturally defined by

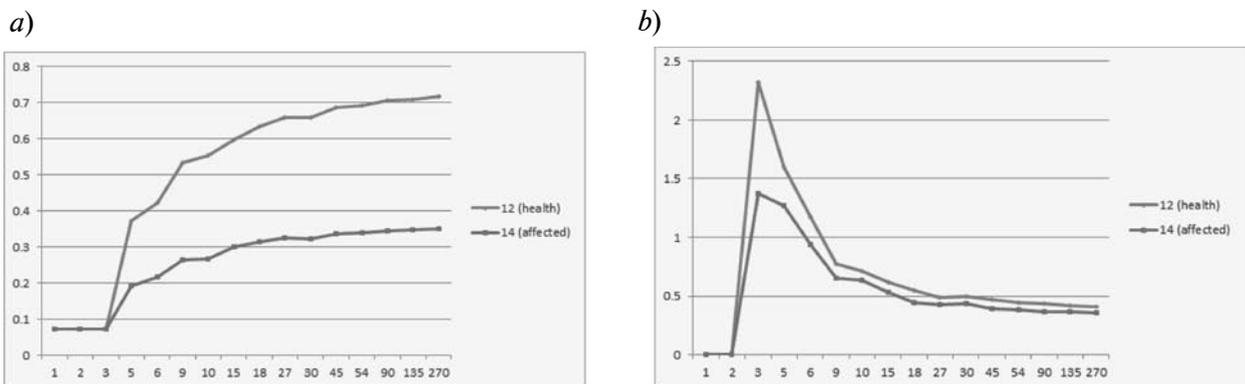


Fig. 2. The dependence the surface area (left) and fractal dimension (right) on cell size for health (number 12) and affected bone tissue (number 14)  
 The image size is 270×270

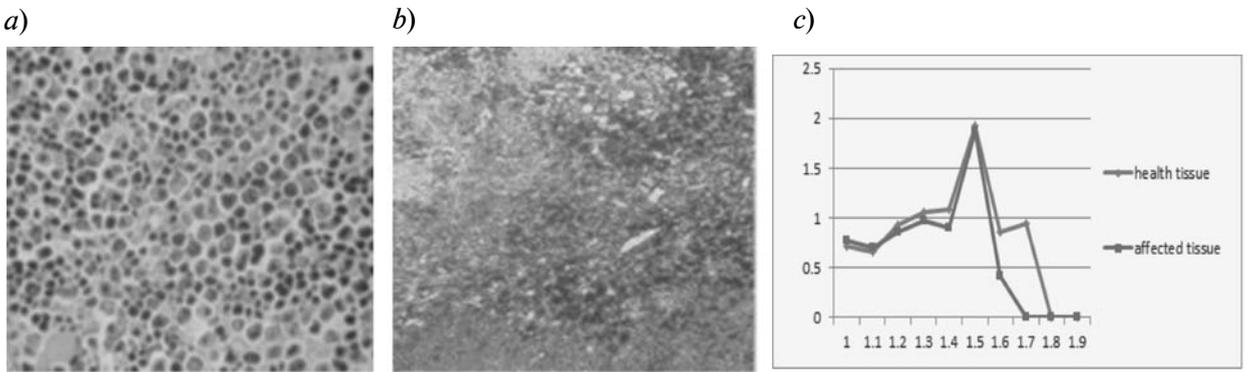


Fig. 3. The health (a) and affected (b) liver preparations and their MFS spectra (c)

pixel intensities [14]). Different categorization methods lead to different multifractal spectra. The method of MFS direct determination was described in [13]. In [8] we applied the method to classify the images of mentioned preparation. It demonstrated reliable results for bone and connective tissues, and was not effective for the images of blood. It is the expected result

because these images do not have any fractal structure. The following illustrations show the images of health and affected liver preparations and the plots of their multifractal spectra (Fig. 3). On the  $OX$  axis the values  $\alpha$  are marked, and the fractal dimensions of corresponding sets  $E_\alpha$  are shown on  $OY$ .

**Classification of images obtained by the**

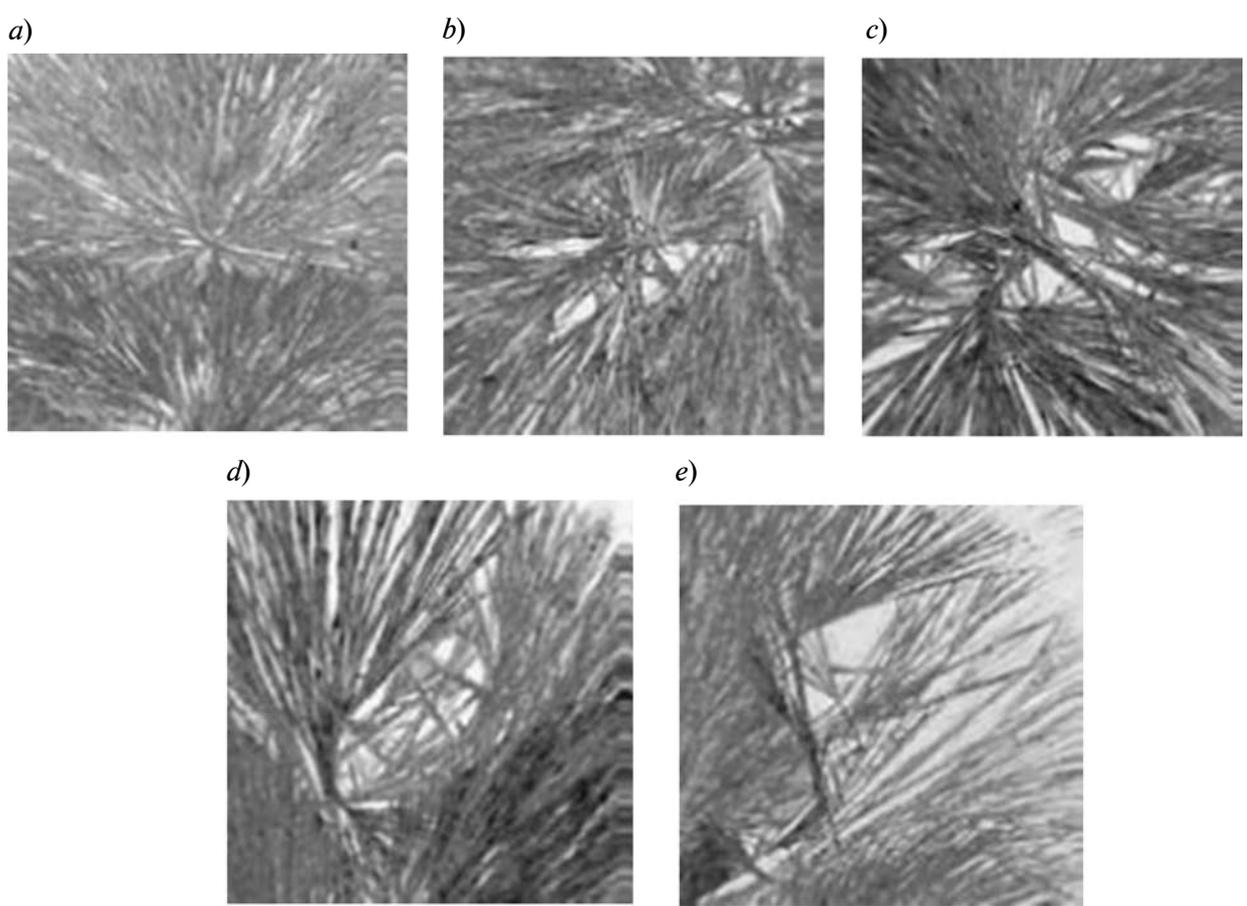


Fig. 4. Patterns of blood crystals obtained by the sensitive crystallization method

**sensitive crystallization method.** As it was mentioned above, the crystals of blood and plant extracts obtained by the sensitive crystallization method are very important in medical diagnosis. Their crystal structures have various forms which may be considered as classification features for classes of preparation. For blood crystals the star form (typical for acute inflammatory process) and the hole structure (typical for degenerative processes) are the examples of structures. The examples of blood crystals are shown on Fig. 4:

crystallization in the star form, which is typical for acute inflammatory process (Fig. 4 *a*);

crystallization in the star form with hole structure, which is typical for chronic inflammatory process (Fig. 4 *b*);

hole structure of crystals, which is typical for degenerative processes (Fig. 4 *c*);

hollow crystal form – typical for benign tumors (Fig. 4 *d*);

hollow crystal form with transversal structures, typical for malignant tumors (Fig. 4 *e*).

Plant extract crystals have their own features. On Fig. 5 typical plant extract crystals from the classes of images used in numerical experiments are shown.

**Statistical textural features.** Spatial gray level co-occurrence estimates image properties related to second-order statistics which considers the relationship among pixels or groups of pixels (usually pairs of pixels). To analyze and classify textures of digital images

Haralick [15] suggested the use of gray level co-occurrence matrices (GLCMs) which have become one of the most well-known and widely used texture features. This method is based on the joint probability distributions of pairs of pixels. GLCM shows how often each gray level occurs at a pixel located at a fixed geometric position relative to each other pixel, as a function of the gray level. The (1,3) entry in a matrix for right neighbors, for example, would show the frequency or probability of finding gray level 3 immediately to the right of pixel with gray level 1.

GLCM has a parameter – a relative position of pixels, which is defined by the angle and distance. To form vectors of features for an image we use the following statistical texture features obtained on a basis of the normalized GLCM –  $P_{i,j}^{\text{Norm}}$ .

Contrast  $\sum_{i,j=0}^{N-1} P_{i,j}^{\text{Norm}} (i - j)^2$  defines a measure of a contrast between a pixel and its neighbours or between a pixel and the whole image.

Homogeneity  $\sum_{i,j=0}^{N-1} (P_{i,j}^{\text{Norm}} / 1 + (i - j)^2)$  describes the density of distribution of elements in  $P_{i,j}^{\text{Norm}}$  relative to its diagonal.

Correlation  $\sum_{i,j=0}^{N-1} P_{i,j}^{\text{Norm}} ((i - \mu I)(j - \mu J) / \sqrt{(\sigma I^2)(\sigma J^2)})$  of the GLCM matrix defines a correlation degree between a pixel and its neighbour or such a degree between a pixel and the whole image. Here  $\mu I$ ,  $\mu J$ ,  $\sigma I^2$  and  $\sigma J^2$

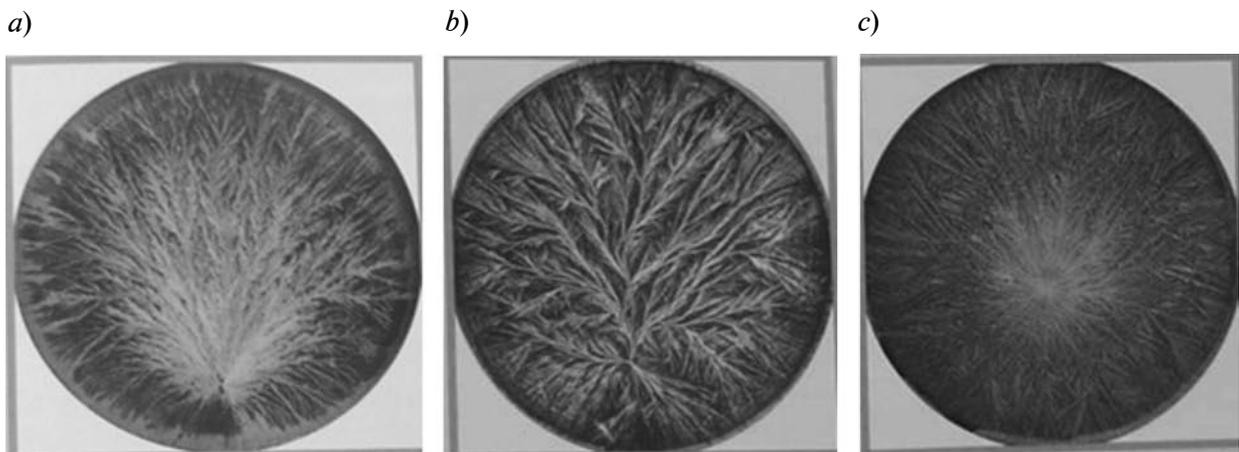


Fig. 5. Patterns of plant extract crystals obtained by the sensitive crystallization method:  
*a* – orange juice; *b* – wheat flour solution; *c* – wheat extract

are mathematical expectations and dispersions of pixel intensities, which are calculated by using a given pixel and its neighbours.

**Textural features based on the Gabor filter.** We define two-dimensional Gabor's function as the following:

$$g(X, Y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{1}{2}\left(\frac{X^2}{\sigma_x^2} + \frac{Y^2}{\sigma_y^2}\right) + 2\pi i\omega x\right)}.$$

Then we construct a Gabor's wavelet basing on the following generating functions:  $g_{mn}(X, Y) = a^{-m} \cdot g(X', Y')$ ,  $X' = a^{-m}(X \cos \theta + Y \sin \theta)$ ,  $Y' = a^{-m}(X \sin \theta + Y \cos \theta)$ ,  $\theta = n\pi/N$ ,  $a > 1$ . Here the integers  $m$  and  $n$  define the scale and orientation of the wavelet respectively:  $m = 0, 1, \dots, M$ ,  $n = 0, 1, \dots, N$ , where  $M$  and  $N$  are the given numbers of scales and orientations.

Let  $I(x, y)$  be a distribution of gray level for a digital image. Define the convolution with the Gabor core  $g_{mn}$ :

$$W_{mn}(x, y) = \int I(u, v) g_{mn}^*(u - x, v - y) dudv.$$

In this formula \* means complex conjugation,  $W_{mn}$  – the result of convolution according to the Gabor core with the scale  $m$  and orientation  $n$ . We suppose that the areas with local textures are spatially homogenous, then the defined below values  $\mu_{mn}$  and  $\sigma_{mn}$  may be considered as the area characteristics for the classification problem:

$$\mu_{mn} = \iint |W_{mn}(x, y)| dx dy.$$

$$\sigma_{mn} = \sqrt{\iint (|W_{mn}(x, y)| - \mu_{mn})^2 dx dy}.$$

The feature vector to classify images is formed as the following  $\vec{f} = [\mu_{00} \ \sigma_{00} \ \mu_{01} \ \sigma_{01} \dots \mu_{mn} \ \sigma_{mn}]$ .

**Morphological features.** Mathematical morphology is a method of obtaining structure components of an image that may be useful to represent and describe it. Boundaries, skeletons and convex hulls are examples of such structures. In this work for the purpose of recognition we use skeletons. This approach leads to reducing the task dimension. To mark out the image skeleton the software packages are used [16].

**The classifier of signs.** In this work three-layered perceptron is used as the classifier. The number of neurons in the input layer of network is defined by the size of a vector of features (depending on the method of the vector construction). The number of neurons in the mean (the second) layer is equal to integer part of one half of the number of neurons in the input layer (an empirical recommendation). In our experiment the number of output neurons (the third layer) is defined by the number of classes of images considered in every group. The algorithm RProp (Resilient Propagation) accelerating the back propagation of error algorithm (Backprop) is applied to setup the network parameters [17, 11].

The classifier testing was performed for three groups of images:

- 1) images of blood crystals obtained by the sensitive crystallization method;
- 2) images of plant extracts crystals obtained by the sensitive crystallization method;
- 3) images of brain tumors.

We considered the following classes of brain tumors – astrocytoma, nevrinoma, oligodendroglioma. As it was shown in [2], for these classes fractal methods give good results. So here we use them to verify the classifier work.

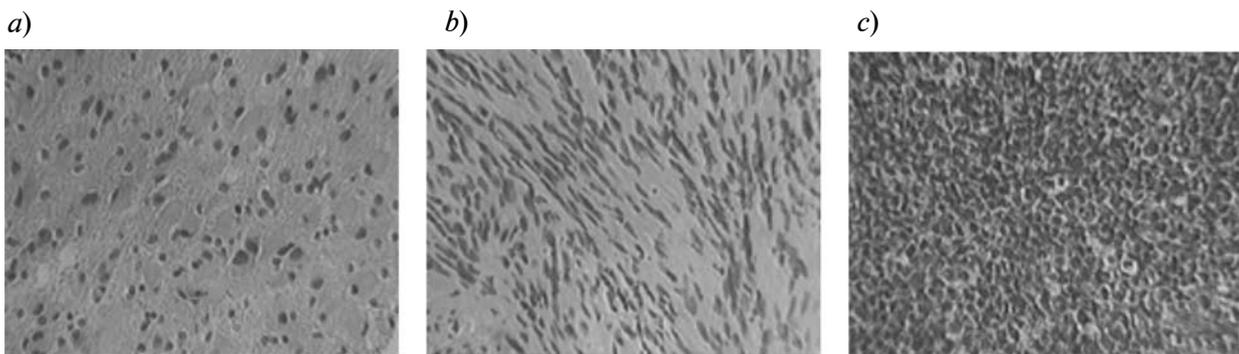


Fig. 6. The representatives of three classes of brain tumors

On Fig. 6 typical representatives of the classes mentioned are shown: *a* – astrocytoma; *b* – nevrinoma; *c* – oligodendrogloma.

For every group a vector of features was calculated on a basis of one of the methods

mentioned above (statistical, spectral and morphological). Then the vector was input to the neural network to construct the classifier. On Fig. 7 the graphics of the dependence mean-square error on the number of iteration

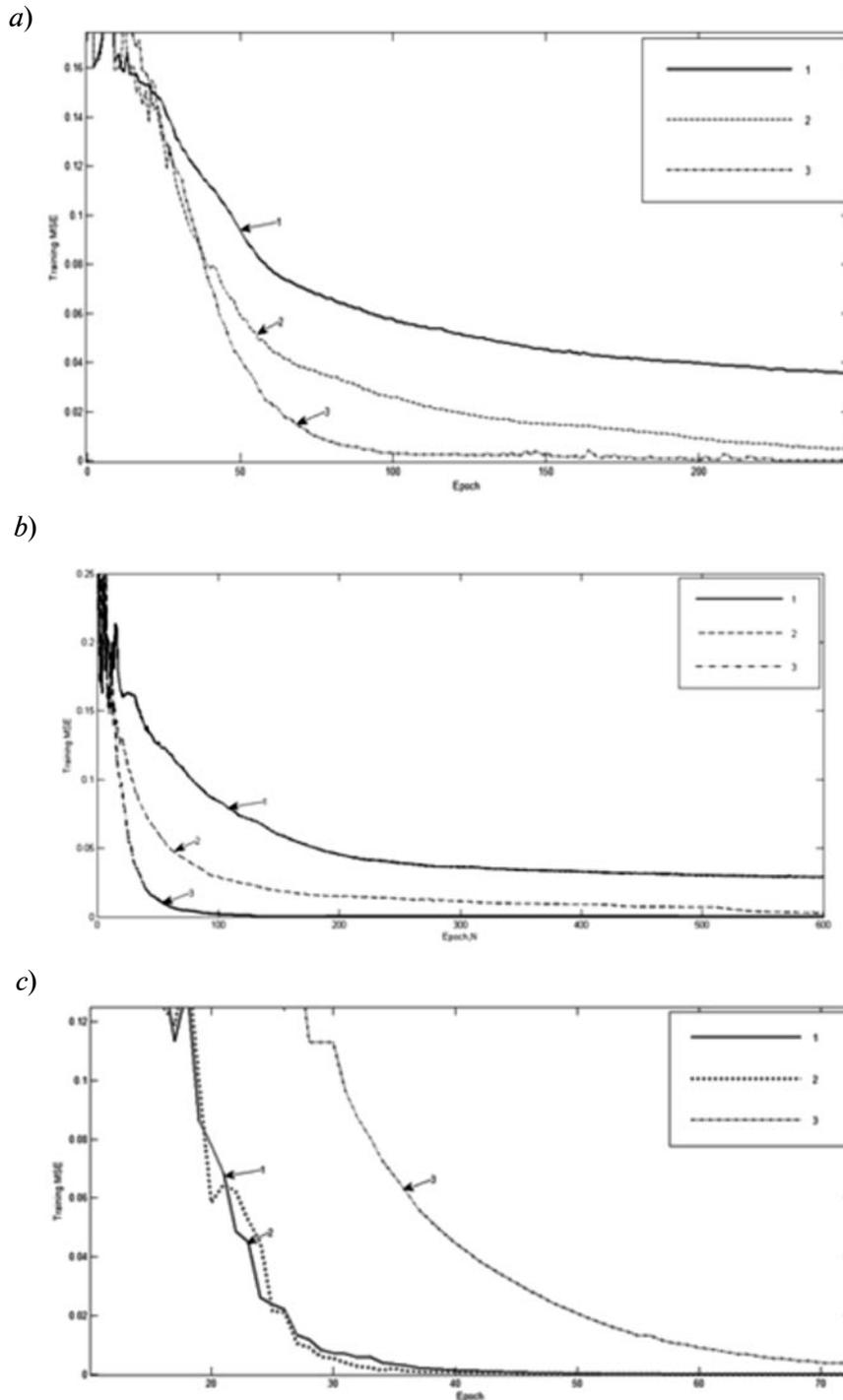


Fig. 7. The graphics of dependence mean-square error on the number of iteration: *a* – for statistical texture features; *b* – for texture features based on Gabor's filter; *c* – for morphological features

in the cycle of weighting coefficients adjusting are shown. The digits 1–3 denote the group of images described above.

The results of experiments show that the model gives good results for images of different classes. The best results (minimal time) for the classification of blood crystal were obtained for morphological signs. For brain tumors statistical signs and Gabor's filters were more preferable. This result is in the conformity with [2], because both the fractal method and statistical characteristics are based on using pixel intensities.

Analysis and interpretation of characteristics obtained on a basis of mathematical

methods are key factors for successful work of specialists in biology and medicine. The investigations made by the authors demonstrated the applicability textural, fractal, spectral and morphological methods to a classification of biomedical preparation images. These results may become a reliable basis to design and implement the tools assisting experts in their practical activity.

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